Cambridge
Intermediate Mathematics

ALGEBRA

PART I

With Answers

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Cambridge Intermediate Mathematics

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PREFACE

The Cambridge Intermediate Mathematics series, consisting of text-books in Arithmetic, Algebra and Geometry, each in two parts, has been designed to meet the needs of pupils in the newly organised Modern Schools and Senior Classes recommended by the Hadow Report.

"The first work of teachers and administrators is," to use the words of *The New Prospect in Education*, just published by the Board of Education, "to think out their goal, feeling their way towards an appropriate curriculum." It is impossible to predict with any certainty the form which the curriculum will ultimately take; it is improbable that any such limitations as are imposed upon the Secondary School can ever be applied to the diversified types of senior schools which are about to spring up. But it is generally agreed that these schools must not become "an anaemic reflection of the present Secondary School."

The underlying notion on which the treatment in the books of this series has been based is that the aim of the Modern Schools, whether selective or non-selective, is to fit the pupils to take their places in the industrial and commercial rather than in the professional walks of life. For the latter the academic course of the Secondary School is a more or less fitting preparation; for the former it is decidedly out of place. It is assumed that the mathematical work of the newly organised schools will have a practical bias; their pupils need to be able to apply principles rather than to be able to derive them. Hence in these books theoretical explanations have been reduced to a minimum, and the use of the results has been emphasised.

ALGEBRA, PART I does not profess to prepare the pupil for any examination, but is intended to serve as a simple introduction to the subject, whether it is taught for the sake of its utility in solving more easily problems in arithmetic, and in applying the simpler formulae, or whether it will be needed eventually for examination purposes. This book, with PART II, will introduce

the pupil to the elementary principles on which the more advanced parts of Algebra are based; if he is transferred at any stage of the course to a Secondary School, or if he takes up his studies again in later life, he will not feel at a loss. PART I will provide a complete course of Algebra for the average non-selective senior school.

Throughout PART I the needs of beginners, and particularly of beginners whose course will probably be concluded at an early age, have been kept in mind. Endeavours have been made to eliminate all sections of Algebra which will not be likely to have any bearing on the easier applications; it will be noticed that involved examples in the use of the four rules have not been included, and until Simultaneous Equations have been learnt, terms of the second and higher degrees have rarely been introduced. It is hoped that, in this way, the beginner's interest will be aroused, since he will be enabled from the outset to see a purpose in the subject.

A feature of the book is the introduction of exercises which can be solved without any working on paper; for want of a better term these exercises have been described as *Mental*. The author believes that there is as much justification for mental algebra as for mental arithmetic, and for the same reason, namely to fix firmly and to revise rapidly the basic principles.

A further feature is the interpolation of a series of sectional revision exercises, each consisting of a *Mental* and a *Written* section; a general revision exercise concludes the book. These two features have also been adopted in the companion volumes of the series.

I am indebted to Mr E. F. Partridge, B.Sc., for his valuable assistance in obtaining solutions to the questions, and the Cambridge Local Examinations Syndicate for permission to use two pages from their *Cambridge Four-Figure Mathematical Tables*.

H. J. L.

September, 1928.

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ALGEBRA: PART I

INTRODUCTION TO SIMPLE EQUATIONS AND PROBLEMS

1. Ask one of your friends to work this puzzle: "Think of a number; double it; add 6 more than the number you thought of; divide by 3. What is the answer?" Having taken away 2, you will give the result as the number first thought of.

The above is a very simple case, but an investigation will help us to find the principles on which such puzzles are based. Suppose 10 is the number chosen:

The given number	10
Doubled	20
6 more than 10 added	36
Divided by 3	12

$12 - 2 = 10$, the number thought of.

This does not prove that the rule is correct in all cases. Let us use x to denote the number thought of:

The given number	x	...............(1),
Doubled	$2x$	...............(2),
6 more than x added	$3x + 6$	...............(3),
Divided by 3	$x + 2$	...............(4).

$x + 2 - 2 = x$, the number thought of.

Notes on the above:

(1) Compare this with 1*s*.; £1; 1*d*. In these cases we always put 1 beside the symbol denoting the unit chosen. We might have used $1x$ to denote 1 of the unit chosen here, but in the new subject to which you are now being introduced, 1 is always omitted before the symbol representing a number.

(2) Doubling x to give $2x$ is very much like doubling 1*s*. to give 2*s*.

(3) This is similar to adding 1*s*. 6*d*. to 2*s*.

(4) Just as 3*s*. 6*d*. $\div 3 =$ 1*s*. 2*d*., so $(3x+6) \div 3 = x+2$. That is to say, we divide both parts by 3.

(5) It is clear that 2 taken from 2 more than a number gives that number as the result.

2. The new subject you are just starting is an extension of arithmetic, using letters to indicate numbers. Algebra, as this subject is called, helps us, amongst other things, to solve quite easily problems which would be difficult to solve if purely arithmetical methods were used.

Consider the following problem:

> "In one pan of a balance I place an object and a 4 oz. weight, and I find that weights amounting to 10 oz. in the other pan just balance these. What is the weight of the object?"

Obviously the answer is 6 oz., but let us obtain it in another way, using x ounces to indicate the weight of the object:

$$x \text{ ounces} + 4 \text{ ounces} = 10 \text{ ounces},$$

or $$x + 4 = 10 \quad \text{.......................(1).}$$

Taking away 4 ounces from each pan we get x ounces in one and 6 ounces in the other, and they balance:

$$\therefore \ x = 6.$$

Note what has been done to the "equation" $x+4=10$. 4 has been taken from each side, or

$$x = 10 - 4 \quad \text{.......................(2).}$$

Comparing equations (1) and (2) we notice that 4 has been taken from the left side and placed on the right *with the sign changed.*

3. Now consider a second example:

> "A bag of sugar is placed in the left pan of a balance and 12 oz. weights in the right. The left pan bumps down, but if 4 oz. of sugar are taken from the bag, the pans just balance. What was the weight of the bag of sugar at first?"

Here again the answer is obvious; viz. 16 oz.

Let x ounces be the weight of the bag of sugar.

Then $x \text{ ounces} - 4 \text{ ounces} = 12 \text{ ounces},$

or $$x - 4 = 12 \quad \text{.......................(1).}$$

Let us now add 4 ounces to the weight in each pan. In the left we shall have the original weight, in the right 16 ounces,

$$\text{or}\quad x - 4 + 4 = 12 + 4,$$

$$\text{or}\quad x = 12 + 4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Note what has been done to the equation $x - 4 = 12$. 4 has been added to each side. Comparing equations (1) and (2) we notice that -4 has been taken from the left side and placed on the right *with the sign changed.* This applies generally to any term changed from one side to the other.

Learn:

Any term can be changed from one side of an equation to the other if the sign also is changed.

4. The following are examples of the rule:

(1) $x + 9 = 18,$
$x = 18 - 9$
$= 9.$

(2) $x - 7 = 10,$
$x = 10 + 7$
$= 17.$

(3) $x - 4 = -2.$
$x = -2 + 4$
$= 2.$

(4) $-x + 6 = 2,$
$6 - 2 = x,$
$4 = x.$

(5) $-x - 2 = -8,$
$8 - 2 = x,$
$6 = x.$

Note what would have happened had we attempted to bring the x terms to the left in examples (4) and (5):

$$-x + 6 = 2, \qquad -x - 2 = -8,$$
$$-x = 2 - 6, \qquad -x = -8 + 2.$$

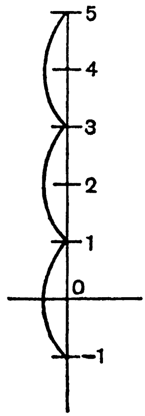
This introduces a difficulty which does not arise in arithmetic. In algebra we say that $2 - 6 = -4$, $-8 + 2 = -6$, etc.

5. Interpretation of a debt.

(1) Suppose I have £100 and owe £150. I am really worth less than nothing, for after paying all I have I still owe £50. We may therefore consider that the value of my cash possessions is £ (-50).

(2) The following example will help to make the use of minus quantities clearer. Let us take the number 5 and keep on taking away 2; we get 5, 3, 1; and from 1 it is impossible to take 2.

From 1 to 0 represents one of the two steps. It becomes convenient to let from 0 to -1 represent the other, as illustrated in the diagram, which shows the series 5, 3, 1, -1, etc.



Similarly the following series of numbers can be developed:

$$15, 10, 5, 0, -5, -10, -15, \text{etc.},$$
$$17, 11, 5, -1, -7, -13.$$

6. Returning to the examples

$$-x+6=2, \qquad -x-2=-8,$$

we have reached the stages

$$-x=2-6 \qquad -x=-8+2$$
$$=-4, \qquad\qquad =-6.$$

Using the rule in para. 3, we can take $-x$ to the right in each case, and the number to the left, and we get

$$4=x, \qquad 6=x,$$
$$\text{or } x=4, \qquad \text{or } x=6.$$

Hence:

The signs of all terms on one side of an equation can be changed if the signs of all terms on the other side are changed also.

E.g. (1) $-x+6=2$, $x-6=-2$. (2) $-x-2=-8$, $x+2=8$.

(3) $-x-4=7$, $x+4=-7$. (4) $-y+3=-2$, $y-3=2$.

7. A balance has been used to illustrate two rules of equations. It will at once be understood that if the two pans balance with certain weights placed in one pan they will balance if only half the weight is used in each, or a third, a quarter, or any other fraction. Similarly they will balance if the weights in both pans are doubled, trebled, or, in fact, if any multiple of both is taken.

Learn:

All terms of an equation may be multiplied or divided by the same quantity without altering the equality.

Ex. 1. $3x = 21$, $x = 7$.

Ex. 2. $3x + 9 = 6x + 12$, $x + 3 = 2x + 4$.

Ex. 3. $\frac{1}{2}x = 8$, $x = 16$.

Ex. 4. $\frac{3}{4}x + 4 = \frac{1}{2}x + 2$, $3x + 16 = 2x + 8$.

Caution! *Every term* on both sides of the equation must be multiplied or divided by the quantity chosen.

8. Sometimes all the above operations must be used, the order depending on the nature of the equation.

Ex. 1. $\frac{2}{3}x - 6 = \frac{1}{6}x + 2$.

First remove fractions by multiplying all terms by 6, the L.C.M. of 3 and 6:

$$6 \times \tfrac{2}{3}x - 6 \times 6 = 6 \times \tfrac{1}{6}x + 6 \times 2,$$
$$4x - 36 = x + 12.$$

Next bring all x terms to the left, and all numbers to the right, taking care to change signs *if sides are changed*, but not otherwise:

$$4x - x = 12 + 36,$$
$$\therefore\ 3x = 48.$$

Divide all terms by 3:

$$x = 16.$$

Ex. 2. $\frac{3}{4} = \frac{2x}{3}$.

Multiply all terms by 12, the L.C.M. of the Denominators:

$$12 \times \frac{3}{4} = 12 \times \frac{2x}{3},$$
$$9 = 8x,$$
$$\text{or}\quad 8x = 9,$$
$$x = 1\tfrac{1}{8}.$$

EXERCISE 1. Mental.

Write down the answers to the following: that is, find the value of x.

1. $x - 1 = 4$.
2. $x + 1 = 3$.
3. $x - 4 = 6$.
4. $x + 1 = 2$.
5. $x - 5 = 0$.
6. $5 - x = 4$.
7. $x + 7 = 5$.
8. $2 - x = 5$.
9. $2 + x = 1$.
10. $2x = 8$.
11. $3x = 9$.
12. $5x = 15$.

13. $7x = 0.$
14. $24 = 8x.$
15. $7x = -14.$
16. $-3x = 9.$
17. $-6x = -24.$
18. $-4x = 8.$
19. $\frac{1}{2}x = 9.$
20. $\frac{1}{3}x = 7.$
21. $\frac{2}{3}x = 0.$
22. $\frac{x}{3} = -4.$
23. $\frac{1}{4}x = 6.$
24. $7\frac{1}{2}x = 0.$
25. $4x - 8 = 0.$
26. $5x - 1 = 0.$
27. $7x + 14 = 0.$
28. $3x + 1 = 10.$
29. $4x - 2 = 8.$
30. $\frac{1}{5}x + 1 = 6.$
31. $\frac{1}{2}x - 1 = 2.$
32. $-3x = 15.$
33. $-4x = 0.$
34. $-2x + 6 = 0.$
35. $-3x = 1\frac{1}{2}.$
36. $\frac{x}{2} = \frac{1}{2}.$
37. $\frac{x}{3} = \frac{1}{6}.$
38. $\frac{x}{4} = -\frac{1}{8}.$
39. $\frac{1}{3}x - \frac{1}{2} = 0.$
40. $\frac{x}{2} = -3.$
41. $-\frac{x}{2} = -\frac{3}{2}.$
42. $\frac{1}{5}x = -5.$

Some Definitions, etc.

9. An expression is often placed within brackets to indicate that the result of the operations within the brackets must be considered as one quantity.

E.g. (1) $10-(4-3)$ means that the *result* of $4-3$ must be taken from 10. Hence this is quite different from $10-4-3$, which means that both 4 and 3 must be taken from 10.

(2) $\frac{2}{3} \div (\frac{1}{2} \times \frac{3}{4})$ means $\frac{2}{3} \div \frac{3}{8}$, for $\frac{1}{2} \times \frac{3}{4}$ has to be considered as one quantity.

Hence this is different from $\frac{2}{3} \div \frac{1}{2} \times \frac{3}{4}$, which means that $\frac{2}{3}$ has to be divided by $\frac{1}{2}$ and that result multiplied by $\frac{3}{4}$.

If letters are placed together in Algebra with no sign between them, or if two quantities within brackets are placed together with no sign between, the symbols or quantities are to be considered as multiplied together; e.g. $3ab$ means $3 \times a \times b$. $(x+3)(x-6)$ means $(x+3)$ times $(x-6)$.

$$x^2 \text{ means } x \times x, \quad x^3 = x \times x \times x, \quad x^4 = x \times x \times x \times x,$$

and so on. Hence $3xy^2 = 3 \times x \times y \times y$.

Terms are said to be "*like*" terms when they differ only in the numerical factor, e.g. $5x$ and $8x$, $7y^2$ and $15y^2$, $3cd^2$ and $-7cd^2$ are pairs of like terms, but $4x^2y$ and $8xy^2$ are "*unlike*" terms.

The "*coefficient*" of a certain symbol in a term is the product of all the other factors of the term; e.g. the coefficient of y in $16y$ is 16; the coefficient of z in $2x^2z$ is $2x^2$.

Like terms can be "*collected*" together by adding their numerical coefficients, taking care of the signs + and −.

E.g. 1. $7x + 5x + 2x = (7 + 5 + 2)x = 14x$.

2. $15a - 2x - 7a + 5x + 5x - 2a$
$= (15 - 7 - 2)a + (-2 + 5 + 5)x$
$= 6a + 8x$.

3. $(3x + 12y + 4z) + (2x - 3y - 3z)$
$= 5x + 9y + z$.

4. The coefficient of x in the expression $5x + 7x$ is $5 + 7$ or 12.

Brackets can be removed without making any other change as long as there is nothing but a positive (or plus) sign before them.

If no sign is put before a term or an expression, the plus sign is understood.

EXERCISE 2. MENTAL.

1. Add together $3x$, $2x$, $5x$, $7x$.
2. From $5a$ take $3a$ and add a to the result.
3. Simplify $32x - 10x$.
4. $(5a + 2b) + (3a + b) + (4a + 3b)$.
5. $5a - 2x - 3a + 4x + 5a + 7x$.
6. $(a + b + c) + (a + b - c) + (a - b + c) + (b + c - a)$.
7. $\cdot 7x + \cdot 5y + \cdot 5x + \cdot 7y$.
8. What is the result when the sum of $(2a + 3b)$ and $(4a - 3b)$ is added to $5a$?
9. From a line x feet long a portion y inches is cut off. How many inches remain?
10. If $X = a + 3b$ and $Y = b + 3a$, find the value of $2X + 3Y$.
11. Add $13x + 2y - 4z$ to $2x + 3y + 13z$.
12. Find only the "x" term in the sum of
$(3x + 4y + 2z)$, $(3y + 4z + 2x)$, $(3z + 4x + 2y)$.
13. Find the value of $12b - 4b$ when $b = 3$.
14. What is the excess of $5x + 7y + 2z$ over $2x + 4y + 2z$?

15. What is the value, when $x=5$, of

$$3x+y-2z+5x+y-3z+2x-2y+5z?$$

16. Add together $\frac{1}{2}x+\frac{1}{3}x+\frac{5}{6}x$.

17. Take $\frac{2}{3}y$ from y.

18. By how much is $3x$ less than 1?

19. Simplify $\cdot 2a+\cdot 3b+\cdot 5a+\cdot 6b+\cdot 3a+\cdot 1b$.

20. Take $3a+4b$ from $5a+4b$.

Substitution.

10. Ex. 1. Find the value of $3x+2y$ when $x=1\cdot 5$, $y=2\cdot 5$.

$$3x+2y=3(1\cdot 5)+2(2\cdot 5)=4\cdot 5+5=9\cdot 5.$$

Ex. 2. Find the value of $2a-3b+4c$ when $a=4$, $b=2$, $c=0$.

$$2a-3b+4c=2(4)-3(2)+4(0)=8-6+0=2.$$

Ex. 3. Show that $x=3$ *satisfies* the equation

$$1\tfrac{3}{5}x-4=1\tfrac{1}{5}x-2\tfrac{4}{5}.$$

Left side $=1\frac{3}{5}x-4$	Right side $=1\frac{1}{5}x-2\frac{4}{5}$
$=1\frac{3}{5}\times 3-4$	$=1\frac{1}{5}\times 3-2\frac{4}{5}$
$=4\frac{4}{5}-4$	$=3\frac{3}{5}-2\frac{4}{5}$
$=\frac{4}{5}$.	$=\frac{4}{5}$.

$\therefore$ Left side = Right side, and the equation is said to be *satisfied.*

Ex. 4. Find the value of $3x^2y+2xy^2$ when $x=1$, $y=2$.

$$\begin{aligned}3x^2y+2xy^2&=3(1)^2 2+2(1)(2)^2\\&=3\times 1\times 2+2\times 1\times 4\\&=6+8\\&=14.\end{aligned}$$

Ex. 5. Find the value of $\dfrac{a^2+b^2+c^2}{ab+ac+bc}$ when $a=2, b=3, c=0$.

$$\begin{aligned}\text{Expression}&=\frac{(2)^2+(3)^2+(0)^2}{2\times 3+2\times 0+3\times 0}\\&=\frac{4+9+0}{6+0+0}=\frac{13}{6}=2\frac{1}{6}.\end{aligned}$$

EXERCISE 3.

Find the values of:

1. $3x+7$ when $x=4$.
2. $15-3x$ when $x=2$.
3. $12+5y$ when $y=-1$.
4. $\dfrac{x+3}{2x+4}$ when $x=0$.
5. $2a+3b+c$ when $a=1$, $b=2$, $c=3$.
6. $3x+4y+2z$ when $x=4$, $y=4$, $z=0$.
7. $x-2y+3z$ when $x=y=z=1$ (i.e. $x=1$, $y=1$, $z=1$).
8. $\cdot5x+\cdot2y+\cdot3z$ when $x=y=z=10$.
9. $3x+7y$ when $x=\cdot5$, $y=1\cdot5$.
10. $5x-y+4z$ when $x=1$, $y=3$, $z=5$.
11. $\dfrac{a+3b-c}{3a-4b+c}$ when $a=5$, $b=3$, $c=0$.
12. $x^3+xy^2+x^2y+y^3$ when $x=2$, $y=3$.
13. $(x+y)^3$ when $x=2$, $y=5$.
14. $(x+2y)(2x+y)$ when $x=5$, $y=1$.
15. $(2a+3b)(3a-b)$ when $a=3$, $b=1$.
16. $\dfrac{4x-y+z}{2x-3y+2z}$ when $x=y=z=4$.
17. $x^2-3ax+2b^2$ when $x=4$, $a=1$, $b=2$.
18. $4abc^2$ when $a=\cdot5$, $b=2$, $c=3$.
19. $axy^2+ax^2y+a^2xy$ when $a=2$, $x=3$, $y=4$.
20. $(a^2+ab+b^2)(a-b)$ when $a=4$, $b=3$.

Show that:

21. $x=2$ satisfies the equation $4x+5=3x+7$.
22. $x=4$ satisfies the equation $2x-1=4x-9$.

11. In the equations which follow it is necessary to *transpose* all the x terms from the right side of the equation to the left, and all the numerical terms from left to right. If a term is so transposed, its sign must be changed. If a term stays on the same side, no change of sign is made.

After the terms have been transposed, the like terms must be *collected* on both sides of the equation:

Ex. 1. $$7x - 5 = 8 - 3x.$$

Transposing, we get

$$7x + 3x = 8 + 5.$$

Collecting,

$$10x = 13,$$

$$x = \tfrac{13}{10} = 1\tfrac{3}{10}.$$

Ex. 2. $$5 - x - 7 = 8 - 5x + 2,$$

$$-x + 5x = 8 + 2 - 5 + 7,$$

$$4x = 12,$$

$$x = 3.$$

Test the answer:

Left side $= 5 - x - 7$

$= 5 - 3 - 7$

$= -5.$

Right side $= 8 - 5x + 2$

$= 8 - 15 + 2$

$= -5.$

EXERCISE 4.

Solve the equations; and test your answers by substitution, as indicated in paragraph 11:

1. $5x - 3 = 17.$
2. $4x - 2 = 14.$
3. $7x + 1 = 15.$
4. $2x + 7 = 13.$
5. $3 + 2x = 13.$
6. $5 + 7x = 19.$
7. $3a + 7 = 2a + 5.$
8. $5y - 2 = 3y - 1.$
9. $5y - 5 = 2y - 7.$
10. $6 - 5x = 4 - 2x.$
11. $3x - 2 - 4x + 3 = 0.$
12. $5x - 7 - 4 + 5x = 3x.$
13. $2y - 7 = 5y - 7.$
14. $4x - 2 = 2 - 4x - 4.$
15. $2x + (7 + 2x) = 3 - x.$
16. $4 + 2x - 3 = 3 + 3x - 4.$
17. $4 - x + 5 - 2x = 6 - 4x.$
18. $5 = 7 - 2x - 3.$
19. $0 = 6 - 3x + 2.$
20. $5 - 2x = -6 + 4x.$
21. $3y + 2 = 5y + 8.$
22. $7 = 2y + 3 - 8y.$
23. $2x = 5 - 2x.$
24. $3 - x - 4 = 2 + 2x - 6.$

Symbolical Expression.

12. Before attempting to solve problems one must first learn how to perform the usual operations with symbols instead of numbers. They will be simple to use if small numbers are substituted for the symbols for the purpose of a preliminary investigation of the principle involved; e.g.

1. Just as 5 more than 4 is $5 + 4$, so 5 more than x is $5 + x$.

2. If you take 7 from 9, you get $9 - 7$. So if you take a from b you get $b - a$.

3. The quotient of 9 divided by 3 is $\frac{9}{3}$. So the quotient of x divided by y is $\frac{x}{y}$.

4. If you are 12 years old now, 4 years ago you were $(12 - 4)$ years old. So if you are x years old now, you were $x - 8$ years old 8 years ago, and you will be $x + 8$ years old in 8 years time.

5. 5, 6 and 7 are "consecutive" numbers, i.e. $5, 5 + 1, 5 + 2$. $\therefore x, x + 1, x + 2$ can stand for any three consecutive numbers. But 5, 6, 7 can also be represented as $6 - 1, 6, 6 + 1$. So also can any three consecutive numbers be represented by $x - 1, x, x + 1$.

6. £5 brought to shillings $= 20 \times 5$ shillings. Also £x brought to shillings is $20x$ shillings. So y lbs. $= 16y$ oz., and b tons $= 20b$ cwts.

7. 3 yds. of cloth at 5*s.* a yard cost 3×5 shillings, so x yds. at 4*s.* a yd. cost $4x$ shillings, and a lbs. of tea at b pence per lb. cost ab pence.

8. If a square has a side of 3 in. the area is 3×3 or 3^2 sq. in. So the area of a square of side x in. is x^2 sq. in.

9. If I travel at 10 miles an hour, I can do 1 mile in $\frac{1}{10}$ hr. In the same way if I travel at x miles an hour, I do 1 mile in $\frac{1}{x}$ hr.

10. A number of two digits such as 47 is equivalent to 4 tens and 7 units, or $10 \times 4 + 7$. Hence any number of 2 digits can be represented by $10x + y$, where x is the ten's digit and y the unit's digit.

These examples are only intended as illustrations. You must think out each fresh case for yourself.

EXERCISE 5. Mental.

1. What number is 3 more than x?

2. Decrease 7 by x.

3. How old will a boy be in x years time if he is 10 years old now?

4. How old was I 10 years ago if I am x years old now?

5. How old shall I be in n years time if I was n years old n years ago?

6. How many pence are there in x shillings?

7. How many shillings are there in £A?

8. Write down three consecutive numbers of which the first is x.

9. Write down three consecutive numbers of which the middle one is x.

10. What is the next odd number greater than $2x+1$?

11. If two numbers differ by 10 and x is the smaller, what is the larger?

12. The sum of two numbers is 24. One of them is x. What is the other?

13. Write down an expression for the perimeter of a rectangle of which one side is l feet and the other b feet.

14. Write down the perimeter of a square of which each side is n inches.

15. What is the cost in shillings of a lbs. of tea at 5*s.* 6*d.* a lb.?

16. If I give A £x, B £3 more than twice this and C £4 more than three times it, how much shall I give them altogether?

17. From £x I spend y shillings and $12z$ pence. How many shillings have I left?

18. If 4 lbs. of sugar cost 1*s.* 4*d.*, what will x lbs. cost?

19. If I travel at the rate of x miles an hour, how many miles is that in a quarter of an hour?

20. If I can row x miles an hour up stream and the rate of the stream is 4 miles an hour, at what rate can I row down stream?

The Solution of Problems.

13. Ex. 1. Find two numbers whose difference is 6 and whose sum is 10.

Let x be one of the numbers.

Since 10 is the sum, the other is $10-x$

and $10-x-x$ is the difference.

$$\therefore 10-x-x=6,$$
$$-2x=6-10=-4,$$
$$\left.\begin{aligned} x&=2\\ 10-x&=8\end{aligned}\right\}.$$

Or we might proceed as follows:

Let x be the smaller number.

Then the larger $=x+6$.

The sum of the two numbers $=x+x+6$.

$$\therefore x+x+6=10,$$
$$2x=4,$$
$$\left.\begin{aligned} x&=2\\ x+6&=8\end{aligned}\right\}.$$

Ex. 2. A father is 3 times as old as his son. In 4 years time the father will be 4 times as old as his son was 2 years ago. Find their ages.

Let $x=$ the son's age now in years,

$3x=$ the father's age,

$3x+4=$ father's age in 4 years,

$x-2=$ son's age 2 years ago.

$\therefore 4x-8=4$ times the son's age 2 years ago.

$$\therefore 3x+4=4x-8,$$
$$3x-4x=-8-4,$$
$$-x=-12,$$
$$\left.\begin{aligned} x&=12\\ 3x&=36\end{aligned}\right\}.$$

Note. *The answers to all problems should be checked.*

EXERCISE 6.

1. If a certain number is added to another twice as great, the result is 24. Find the number.

2. If 4 is taken away from three times a certain number, an answer 14 is obtained. What is the number?

3. A boy is 7 years old. In x years time he will be 20. Find x.

4. If I add together the same number of shillings as of pence, the result is 13 shillings. How many of each do I take?

5. One side of a rectangle is 4 inches longer than the other, and the perimeter is 2 feet. Find the shorter side.

6. The sum of two numbers is 13 and one of them is 3 more than the other. Find them.

7. The sum of three consecutive numbers is 48. Find the middle number.

8. A man buys 8 lbs. of tea at 1*s.* a lb. and some more at 2*s.* a lb. He sells it at 1*s.* 4*d.* a lb. and makes no profit. How much did he buy at 2*s.* a lb.?

9. Two numbers differ by 10 and three times the smaller equals twice the larger. Find the numbers.

10. If I take a certain number from 10 I get the same result as if I take three times the number from 30. Find the number.

11. A father is three times as old as his son. In 20 years time he will only be twice as old. How old is his son now?

12. The sum of two numbers is 45 and one of them is 5 more than the other. Find them.

13. If I had four times as much money as I have and spent 5 shillings, I should have twice as much left as I actually had to begin with. How much had I?

14. Divide £25 between A and B so that A gets £4 more than twice as much as B.

15. A certain number of pounds, twice as many shillings and three times as many pence come to £16. 13*s.* 9*d.* How many of each are there?

16. Someone said to me "Think of a number. Double it. Add 7. Take away 3 more than the number thought of." I gave an answer 39. What number was I thinking of?

17. A boy is 6 years younger than another and in four years time he is half as old as the other. How old are they now?

18. Divide £35 among A, B and C, giving B twice as much as A and C twice as much as B.

19. Divide a line 15 inches long into two parts so that 6 inches more than the longer part is twice as much as the shorter.

20. In a school of 600 children x are boys, twice as many as this are infants and there are 40 more girls than boys. Find the number of each.

Simple Fractional Equations.

14. You have learnt that both sides of an equation may be multiplied by the same number without altering the value. This rule is useful in solving fractional equations, for if each term is multiplied *by the* L.C.M. *of the denominators* all fractions will be removed.

Ex. 1.
$$\frac{3x}{4} - \frac{2x}{6} = 10.$$
$$\text{L.C.M.} = 12.$$
$$12 \times \frac{3x}{4} - 12 \times \frac{2x}{6} = 12 \times 10.$$
$$9x - 4x = 120.$$
$$5x = 120,$$
$$x = 24.$$

Ex. 2.
$$\cdot 5x + 3 = \cdot 7x + 9.$$
Multiply throughout by 10.
$$5x + 30 = 7x + 90,$$
$$5x - 7x = 90 - 30,$$
$$-2x = 60,$$
$$x = -30.$$

Ex. 3.
$$\frac{5}{x} + \frac{3}{6x} = \frac{7}{12x} - 5.$$
$$\text{L.C.M.} = 12x.$$
$$60 + 6 = 7 - 60x,$$
$$60x = -59,$$
$$x = -\frac{59}{60}.$$

Note carefully that *every* term must be multiplied by the L.C.M. of the denominators.

EXERCISE 7.

Solve the following equations, and test each answer by substituting the value of the unknown quantity in the original equation:

1. $\frac{x}{2}-\frac{x}{3}=1.$
2. $\frac{3}{4}z+1=\frac{1}{4}z+5.$
3. $\frac{4m}{5}=\frac{8}{15}.$
4. $\frac{1}{2}x+1\frac{1}{3}=x+\frac{2}{3}.$
5. $2x-1=1\frac{2}{3}x-\frac{1}{2}.$
6. $\frac{3x}{2}-\frac{3x}{5}=9.$
7. $\cdot5x+\cdot3=\cdot6x+\cdot7.$
8. $1\cdot5x-3x=4\cdot5-2\cdot5x.$
9. $\frac{x}{2}+\frac{x}{3}-\frac{x}{6}=4.$
10. $\frac{x}{4}+\frac{x}{5}-\frac{x}{20}=4.$
11. $\cdot8x+\cdot3x=22.$
12. $\cdot7x-\cdot3x=\cdot5-\cdot2x.$
13. $3-\frac{3x}{4}=2-\frac{2x}{3}.$
14. $\frac{3}{8}x-\cdot6=\frac{1}{8}x-\cdot3.$
15. $\frac{5x}{6}=\frac{3x}{8}+1.$
16. $\cdot17x=3-\cdot13x.$
17. $0=\frac{2}{3}-\frac{4x}{6}-\frac{5x}{9}.$
18. $\frac{225}{4x}=\frac{15}{4}.$
19. $\frac{360}{x}=\frac{15}{2}.$
20. $\frac{15x}{32}=\frac{x}{8}+4.$

Removing Brackets.

15. Consider the example $9-(5+2)$.

This means that the sum of 5 and 2 is to be taken from 9, though the same result would be obtained by subtracting them separately; for $9-5=4$ and $4-2=2$, whilst $9-7$ is also 2.

We might write down the operation as follows:

$$9-(5+2)=9-5-2.$$

Hence in removing the brackets, which have a minus sign before them, we *change the signs of the terms*. Needless to say, if a plus sign precedes the brackets the latter are removed without any change of sign,

$$9+(5+2)=9+5+2.$$

Next consider $9-(5-2)$. This means that if 5 were taken from 9 we should have taken away 2 too many, for we need only take 3 from 9 to get the correct answer.

$$\therefore 9-(5-2)=9-5+2.$$

Here again all signs are changed on removing brackets; whereas

$$9+(5-2)=9+5-2.$$

Finally we have the only difficult case, viz. $9-(2-5)$.

For the present it is sufficient for you to accept the rule given above, viz. change all signs on removing the brackets, because they are preceded by a minus sign.

$$\therefore 9-(2-5)=9-2+5.$$

If a proof is needed, the following may be sufficiently convincing.

Let $$9-(2-5)=x \qquad\text{.......................(1).}$$

Take $-(2-5)$ to the other side, changing the sign,

$$9=x+(2-5)=x+2-5.$$

Now take the 2 and -5 back again, changing the signs,

$$9-2+5=x \qquad\text{.......................(2).}$$

From (1) and (2),

$$\therefore 9-(2-5)=9-2+5.$$

Hence we have the following rules for all cases of the removal of brackets.

1. *If brackets are preceded by a plus sign, simply remove the brackets.*

2. *If brackets are preceded by a minus sign, the signs of terms within brackets must be changed on removing the brackets.*

EXERCISE 8.

Simplify by removing brackets and then collecting up like terms:

1. $15+(7-3)$.
2. $7-(2+5)$.
3. $2a+(3a-a)$.
4. $3x-(x-2)$.
5. $5x-(2x-3)$.
6. $3a-2b-(2a+3b)$.
7. $5x-2y-(5x+2y)$.
8. $2c+3d+e-(2c+3d-e)$.
9. $(a-2b)-(2a-b)-(a-2b)$.

10. $3p-5q-(5p-2q)-(3p-2q)$.
11. $5x+7+(3x-2)-(3x+2)$.
12. $s-(3s-t)-t$.
13. $4a+3b+(2b+3c)-(3b+4c)+(2a+3b)$.
14. $(2x+3y+z)-(x+y)-(x+z)-(y+z)$.
15. $2x+7y-(x+2z)-(y+2z)+(x+2y)$.
16. $1{\cdot}5x-2-({\cdot}5x+3)-({\cdot}5x-2)-({\cdot}5x-1)$.
17. $1-({\cdot}5x+{\cdot}2y)$.
18. $0-(x-2y)-(x+2z)+(y-2z)$.
19. $5-(3+2x-3y)+(2+3x-3y)$.
20. $3a+2b+4c-(3a+2b+4c)$.

16. Sometimes a number or a symbol comes before the brackets, and in such cases *every* term within the brackets has to be multiplied by it.

E.g. $6(8+5)=6\times 8+6\times 5=48+30=78$.

No proof should be needed, though it is possible to prove the rule quite easily:

$$2(6+5)=(6+5)+(6+5)=6+5+6+5=2\times 6+2\times 5,$$

$$\begin{aligned}3(7-1)&=(7-1)+(7-1)+(7-1)=7-1+7-1+7-1\\&=3\times 7-3\times 1.\end{aligned}$$

When the brackets are preceded by a minus quantity it is necessary to change signs as before, as well as multiplying by the quantity:

$$\text{E.g.}\quad \begin{aligned}-3(2x-3)&=-6x+9,\\-3(2x+3)&=-6x-9.\end{aligned}$$

EXERCISE 9.

Simplify:

1. $2(3+x)$. 2. $3(3-2x)$. 3. $5(5x-2y)$.
4. $-3(x+3)$. 5. $-2(2x-4)$. 6. $-3(3x-6)$.
7. $3(x-2)+2(x-3)$. 8. $4(2x-1)+3(3x-2)$.
9. $5(5-x)-3(2+x)$. 10. $2(2x+3)-3(3x+2)$.
11. ${\cdot}5(1-x)-{\cdot}25(2-x)$. 12. $1-3(x-\frac{1}{3}y)$.

13. $2\left(1-\frac{x}{2}\right)-3\left(1-\frac{x}{3}\right)$. 14. $6\left(3-\frac{x}{3}\right)-4\left(2+\frac{x}{2}\right)$.

15. $5(q+2r)+2(q+3r)+4(q+4r)$.

16. $3(m-2n)-2(m-5n)-4(m-3n)$.

17. $5(x+2y+z)-(x-2y+z)$.

18. $5(s+t)-2(s+r)-3(s+t)-4(t+r)$.

19. $2a-3b-2(2a-4b)+3(3a-b)$.

20. $5x+7-2(2x+3)+3(4x-6)$.

EXERCISE 10.

Solve the equations:

1. $3(x-2)+2(x+3)=10$.
2. $5(x-1)-3(x-2)=15$.
3. $2(x-3)=3(x-2)$.
4. $2x-7=4(x-5)$.
5. $0=3(x-2)+2(x-3)$.
6. $7(2x+4)+2(3x-2)-2(8x-5)=0$.
7. $5(x+1)-2(x+2)-(x-5)=5$.
8. $4=3q-7-(2q-5)$.
9. $6(5-2x)-3x+5=7(2-x)$.
10. $2x-5=2(x-2)+3(x-3)$.
11. $5x-5(x-1)=2x-4(x-2)$.
12. $2a-3=3(a-2)+4(a-3)-(a-4)$.
13. $5x-2-(3x+1)=4x-3-(3x-1)$.
14. $4(x-1)-5(x-2)=6(x-3)-3(x-4)$.
15. $2x-3+3(x-2)+4(x-3)=5(x-5)$.
16. $2=3(x-2)-2(x-3)+4(x-1)$.
17. $4(x+1)=3x-2(2x-6)$.
18. $2x-3=2(2x-3)$.
19. $3(x-1)=4(x-2)-2(x-3)$.
20. $5(x-2)+4(x-3)=3(x-4)-2(x-5)$.

Simple Fractions involving the use of brackets.

17. Ex. 1. Consider $\dfrac{3x-2}{10}-\dfrac{x-1}{12}$.

As usual, find the L.C.M. of denominators, viz. 60. Bring each fraction to this denominator by multiplying top and bottom by a suitable number.

$$\text{Here the expression} = \frac{6(3x-2)}{6\times 10}-\frac{5(x-1)}{5\times 12} \quad\text{..............(1)}$$
$$=\frac{6(3x-2)-5(x-1)}{60} \quad\text{..............(2)}$$
$$=\frac{18x-12-5x+5}{60}$$
$$=\frac{13x-7}{60}$$

In practice, we usually omit line (1) and proceed to line (2) as follows:

$$60\div 10=6,\ \therefore\ 6(3x-2);$$
$$60\div 12=5,\ \therefore\ -5(x-1).$$

Ex. 2. $\dfrac{2x-1}{3}-\dfrac{3x-1}{4}-\dfrac{4x-1}{6}-\dfrac{3x-2}{12}$.

L.C.D. $=12$.

In practice we proceed as follows:

$$12\div 3=4,\ \therefore\ 4(2x-1);$$
$$12\div 4=3,\ \therefore\ -3(3x-1);$$

and so on.

$$\text{Expression}=\frac{4(2x-1)-3(3x-1)-2(4x-1)-(3x-2)}{12}$$
$$=\frac{8x-4-9x+3-8x+2-3x+2}{12}$$
$$=\frac{-12x+3}{12}$$
$$=\frac{3(-4x+1)}{12}=\frac{1-4x}{4}.$$

Note that $1-4x$ is preferable to $-4x+1$.

Ex. 3. $$\frac{2(2x-1)}{4}+\frac{3(3x-1)}{5}-x+2.$$

Note carefully how we deal with $-x+2$.

$$\text{Expression}=\frac{10(2x-1)+12(3x-1)-20x+40}{20}$$

$$=\frac{20x-10+36x-12-20x+40}{20}$$

$$=\frac{36x+18}{20}=\frac{18(2x+1)}{20}=\frac{9(2x+1)}{10}.$$

EXERCISE 11.

Simplify:

1. $\dfrac{2x-3}{4}+\dfrac{x-2}{3}$.

2. $\dfrac{x-1}{2}-\dfrac{2x-1}{4}$.

3. $\dfrac{5x-1}{4}-\dfrac{2x-1}{6}$.

4. $\dfrac{2x+1}{5}+\dfrac{x+2}{10}$.

5. $\dfrac{1}{2}+\dfrac{3x+1}{4}+\dfrac{2x-3}{6}$.

6. $\dfrac{2}{3}+\dfrac{x+2}{12}-\dfrac{3x-1}{6}$.

7. $\dfrac{3}{4}+\dfrac{x-1}{6}-\dfrac{2x-1}{12}$.

8. $\cdot 75+\cdot 5(x-2)+\cdot 25(x-4)$.

9. $\cdot 6+\cdot 3(2x-1)-\cdot 4(4x-3)$.

10. $\cdot 15+\cdot 05(2x-1)-\cdot 35(4x-1)$.

11. $\dfrac{x}{2}+\dfrac{2x}{3}+\dfrac{3x}{4}-\dfrac{2x-1}{6}$.

12. $\dfrac{2x}{3}-\dfrac{3x}{4}-\dfrac{x-1}{12}+\dfrac{2x-3}{6}$.

13. $\dfrac{x-1}{2}-\dfrac{x-2}{3}-\dfrac{x-3}{4}-\dfrac{x-5}{6}$.

14. $\frac{2}{3}(x-2)-\frac{3}{4}(x-3)-\frac{1}{2}(x-5)$.

15. $\frac{3}{8}(2x-4)+\frac{1}{8}(3x-2)-\frac{7}{8}(x-3)$.

16. $\frac{4}{5}(3x-2)-\frac{1}{5}(4x-3)-\frac{3}{10}(x-2)$.

By simplifying separately the two sides, show that:

17. $\dfrac{x-2}{3}-\dfrac{x-3}{4}=\dfrac{x-1}{6}-\dfrac{x-3}{12}$.

18. $\frac{4x-5}{10}-\frac{x-1}{2}=\frac{3x+1}{5}-\frac{7x+2}{10}$.

19. $\frac{x}{2}-\frac{x-1}{3}=\frac{x+1}{6}+\frac{1}{6}$.

20. $\frac{3(3x-2)}{4}+\frac{2(2x-3)}{6}=2(x-1)+\frac{11x-6}{12}$.

EXERCISE 12.

Solve the equations:

1. $\frac{2x-3}{4}=\frac{3x-2}{5}$.

2. $\frac{x-1}{2}=\frac{3x-2}{4}$.

3. $\frac{1}{4}(3x-5)=\frac{1}{3}(2x-3)$.

4. $\frac{1}{5}(2x-1)=\frac{1}{6}(x-2)$.

5. $\frac{3x-1}{4}=\frac{2x-1}{5}+1$.

6. $\frac{3x-7}{4}=1-\frac{2x-3}{2}$.

7. $\frac{x-1}{2}-\frac{x-2}{3}=4$.

8. $\frac{2x-1}{4}+\frac{2x-3}{3}=5$.

9. $3(x-\frac{2}{5})+4(x-\frac{3}{5})=10\frac{2}{5}$.

10. $\frac{2(x-3)}{4}-\frac{3(2x-1)}{6}+x+1=0$.

11. $\cdot5(2x-3)+\cdot75(4x-6)=5$.

12. $\frac{2}{3}(x-2)-\frac{3}{4}(x-3)=0$.

13. $\frac{3x-1}{6}=\frac{x+2}{12}+\frac{2}{3}$.

14. $\frac{4x-5}{10}-\frac{x-1}{2}=\frac{2x+3}{5}$.

15. $\frac{1}{2}(x-2)-\frac{1}{3}(x-3)=\frac{1}{4}(x-4)$.

16. $\frac{5x-6}{6}-2=\frac{3x-5}{5}-3$.

17. $1-\frac{x-5}{12}=2-\frac{x-4}{6}$.

18. $\cdot5(2-4x)=\cdot75(3-5x)$.

19. $\frac{3x-7}{5}-\frac{2x-3}{3}=\frac{4x-5}{12}$.

20. $\frac{x}{5} - \frac{1}{3}(2x-1) = 4 - \frac{2}{3}(2-x)$.

21. $\frac{20}{x} - \frac{24}{2x} = 2$.

22. $\frac{1}{x} - \frac{2}{3x} = 5$.

23. $\frac{2}{5x} + \frac{7}{10x} = \frac{1}{x} - 2$.

24. $\frac{7}{x} - \frac{3}{2x} = \frac{1}{5x} + 3$.

25. $\frac{1}{2x} - \frac{2}{3x} = \frac{4}{6x} + 1$.

More difficult problems.

18. Ex. 1. Divide £100 between A and C so that A gets £10 more than half of what C gets.

Let C have £x.

$$\text{Half } C\text{'s share} = \frac{x}{2},$$

$$A\text{'s share} = \frac{x}{2} + 10,$$

$$\therefore x + \frac{x}{2} + 10 = 100,$$

$$x + \frac{x}{2} = 90,$$

$$2x + x = 180,$$

$$3x = 180,$$

$$x = 60,$$

$\therefore$ C gets £60 and A £40.

Ex. 2. If I buy a certain number of eggs at 7 a shilling and sell them at 2*d.* each I gain 5*s.* How many do I buy?

Let $x =$ the number of eggs bought.

7 cost 12*d.*, $\therefore$ x cost $\frac{12x}{7}$ pence.

1 is sold for 2*d.*, $\therefore$ x are sold for $2x$ pence.

$$\text{Gain} = 2x - \frac{12x}{7},$$

$$\therefore 2x - \frac{12x}{7} = 60.$$

Note carefully that *all* terms here are expressed in *pence*.

$$14x - 12x = 420,$$
$$2x = 420,$$
$$x = 210.$$
$$\therefore \text{ I buy } 210 \text{ eggs.}$$

Ex. 3. A father is twice as old as one son and three times as old as the second. Six years ago his age was half as much again as the total of the sons' ages. How old is each now?

Ages now.	Ages 6 years ago.
Let x years = father's age.	$x - 6.$
$\frac{x}{2}$ = 1st son's age.	$\frac{x}{2} - 6.$
$\frac{x}{3}$ = 2nd son's age.	$\frac{x}{3} - 6.$

Total of sons' ages 6 years ago $= \frac{x}{2} - 6 + \frac{x}{3} - 6.$

$$\therefore x - 6 = 1\tfrac{1}{2}\left(\frac{x}{2} - 6 + \frac{x}{3} - 6\right).$$
$$2x - 12 = 3\left(\tfrac{5}{6}x - 12\right),$$
$$2x - 12 = \tfrac{5}{2}x - 36,$$
$$4x - 24 = 5x - 72,$$
$$-x = -48,$$
$$x = 48, \text{ the father's age in years.}$$
$$\frac{x}{2} = 24, \text{ the 1st son's age.}$$
$$\frac{x}{3} = 16, \text{ the 2nd son's age.}$$

A caution! At the stage $x - 6 = 1\frac{1}{2}\left(\frac{x}{2} - 6 + \frac{x}{3} - 6\right)$, we multiplied both sides by 2. The left side clearly becomes $2x - 12$, but many beginners proceed to multiply both $1\frac{1}{2}$ and $\left(\frac{x}{2} - 6 + \frac{x}{3} - 6\right)$ by 2. This is incorrect: it is just as bad as saying that if $6 = 3 \times 2$,

$$\therefore 6 \times 4 = (3 \times 4) \times (2 \times 4),$$

though it would be quite correct to say

$$6 \times 4 = (3 \times 4) \times 2.$$

EXERCISE 13.

1. Find a number of which the half is greater than the third part by 25.

2. The sum of the fourth, fifth and sixth parts of a certain number is 740. Find the number.

3. A, B and C share £17 so that A gets half as much again as B and B £3 less than C. How much does A get? [Let £x = C's share.]

4. I divide three consecutive numbers by 7, 5 and 2 respectively, and the sum of these three quotients is 13. Find the numbers.

5. A is half as old as B. In five years B will be three times as old as A was 5 years ago. How old is each now?

6. I bought a house and gained £200 in selling it. If I had paid 10 °/ₒ more than I did, I should have gained nothing in selling. What was my purchase price?

7. Divide £56 into three parts so that $\frac{1}{6}$ of the greatest, $\frac{1}{5}$ of the next, and $\frac{1}{3}$ of the smallest are exactly the same.

8. A travels at 12 miles an hour and B at 16 miles an hour. They start at the same time and ride towards one another from places 63 miles apart. How far are they from these places when they meet?

9. From a basket of eggs A takes $\frac{1}{3}$ and one more. B puts in twice as many as there were at first and there are then 71 eggs in the basket. How many were there at first?

10. I have £1. 13*s*. 0*d*. in half-crowns and florins, and there are 6 more half-crowns than florins. How many are there of each?

11. A father is three times as old as his son. Eleven years ago he was five times as old. What are their present ages?

12. In a railway carriage 6 people can sit on each side. When there are only 5 on each side, each person has two inches more room than if the carriage was full. Find the length of each seat.

13. In an election $\frac{3}{5}$ of the voters voted for one candidate. If 320 of these had voted for the other, the result would have been a tie. How many voted for each? [Let $5x$ = the total number of voters.]

14. Divide 160 into three parts so that the smallest is one-quarter of the largest and one-third of the middle number.

15. Find two numbers whose difference is 12, if one-seventh of the smaller taken from one-sixth of the larger leaves 3.

16. A man has 10 less cows than horses. If he sells half his horses and one-third of his cows, he has the same number of each. How many cows had he at first?

17. A has three times as much money as B. He gives half his money to B and then B has £20 more than A. How much had A at first?

18. £60 is divided among A, B and C. A has £10 more than half as much as B, and B £10 more than half as much as C. How much does each get?

19. Of a certain sum of money I give one-quarter to B and one-fifth of the remainder to C. I then have £120 left. What had I at first?

20. A tradesman bought a number of articles. He sold them so as to gain 3*d.* each on 240 of them and lost 2*d.* each on the rest. Altogether he gained 10*s.* How many did he buy?

SECTIONAL REVISION A

EXERCISE 14 (*a*). MENTAL

Write down the answers without showing any working.

1. Simplify $-2(3-4x)$.
2. Find the value of $3x+2y$ when $x=2$, $y=-3$.
3. From $3a+5b$ take the sum of $a+b$ and $a+2b$.
4. Simplify $\frac{x}{3}+\frac{x}{4}$.
5. Solve the equation $\frac{t}{4}=\frac{1}{2}$.
6. How much greater is $3a$ than $2b$?
7. Add together $4m+2n$, $3m-2n$, $5m+4n$.
8. Bring $\frac{5q-7}{15}$ to a denominator 30.
9. Simplify $\frac{4a-8}{12}$.
10. Write down an equation for the solution of the following problem:

"Divide 27 into two parts so that the second is three times one more than the first."

EXERCISE 14 (*b*).

1. Solve the equation $\frac{1}{2}x-2=\frac{1}{3}x+2$.
2. What is the coefficient of x in the difference of $7x+2y-3$ and $2x+5y$?
3. Add together $\frac{1}{3}m+\frac{1}{4}n$, $\frac{1}{2}m+\frac{1}{2}n$, $\frac{1}{4}m-\frac{1}{3}n$.
4. Find the value of q when $10-3q-4=4-2q-8$.
5. How old shall I be in x years time if I was y years old 5 years ago?
6. Write down a table of pairs of values for x and y when $x=-2, -1, 0, 1, 2$ if $7x+y=36$.
7. Solve for v: $\frac{4v}{5}-\frac{3v}{4}=7-\frac{3v}{10}$.
8. Simplify $5(x-3y)-2(x-3z)-4(y-3z)$.

9. Solve the equation $\frac{5x-3}{2} - \frac{x-3}{4} = \frac{x-1}{5}$.

10. I can row 4 miles an hour up stream and 6 miles an hour down. What is the rate of the stream, and how fast can I row in still water?

EXERCISE 15 (*a*). Mental

Write down the answers without showing any working.

1. Add together $2(a+b)+3(a+2b)$.
2. Simplify $3(l-2m)-5(n-2p)$.
3. If $\frac{2x}{3} = \frac{3x}{2}$, find x.
4. Simplify $\frac{3x}{5} - \frac{7y}{10}$.
5. Solve $5a-7=7-2a$.
6. Find the value of $4(3-2x)$ when $x = \cdot 75$.
7. Take $a+2b+3c$ from $3a+2b+4c$.
8. If $256x = 512$, what is x?
9. If I buy eggs at 7 for a shilling, how many shall I get for £x?
10. Write down the equation by means of which this problem can be solved:

"A train travels twice as fast from B to C as from A to B. If the distances are each 100 miles and the total journey is done in 5 hours, find the speed from A to B."

EXERCISE 15 (*b*).

1. By simplifying separately the two sides, prove that

$$3(a-2b)+4(a-2c)+(a+14b+16c)=8(a+b+c).$$

2. Solve the equation $\frac{2}{3}x + 1\frac{1}{2} = \frac{1}{3}x + 2\frac{1}{2}$.
3. I have £x and spend $20y$ shillings and $120z$ pence. How many shillings have I left?
4. If $2x-y=19$, find y when $x=2, 3, 4, 5, 6$, and write down the table of corresponding values of x and y.
5. Find r when $\frac{2}{3}r - \frac{5}{6} = \frac{5}{6}r - 4\frac{5}{6}$.
6. Simplify $5x-2y+z-2(2x-3y+4z)-(x+z)$.

7. Solve the equation $\frac{4(s-3)}{5} - \frac{3(s-1)}{10} = \frac{3(s-2)}{4}$.

8. Simplify $\frac{5}{6} + \frac{2x-1}{3} - \frac{1-2x}{12}$.

9. Find the coefficient of x in the difference of $2x-3y+4$ and $5x-2y+4x$.

10. Divide 35 into two parts so that the first is three-quarters of the second.

EXERCISE 16 (*a*). MENTAL

Write down the answers without showing any working.

1. Solve the equation $5r-2+2r+3=5r+7$.

2. Simplify $3(l-2m)-2(n-3q)$.

3. If $\frac{5x}{12}=\frac{3}{4}$, find x.

4. What is the coefficient of x in the sum of

$$(5x+7y+4z),\ (2y+4z+13x),\ (z-2y-9x)?$$

5. Find the excess of $5y$ over 1.

6. If $5x+y=17$, find y when $x=3$.

7. Find the value of $7q-3r$ when $q=r=2$.

8. Take $2x+3y$ from $4x+9y$.

9. A man walks x miles due North, then y miles due North, and finally z miles due South. How far is he North of his starting point?

10. Write down an equation which will solve the following problem:

> "Find two numbers of which the sum is 29, such that one-third of the greater is 2 less than half the smaller."

EXERCISE 16 (*b*).

1. Add together $(\cdot 5x+\cdot 25y)$, $(\cdot 75x+\cdot 5y)$, $(\cdot 25x+\cdot 75y)$.

2. Write down 5 consecutive odd numbers of which the middle one is $4x+1$.

3. If x is given the values 0, 1, 2, 3, 4, 5, find the values for y when $5x+2y=26$.

4. What is the value of t if $\frac{3t}{7}+\frac{1}{2}(t-2)=\frac{t+34}{4}$?

5. Simplify $2-3(a+2b-4c)+4(a-3b+2c)$.

6. Solve for r: $\frac{7(3-r)}{2}=\frac{5(6-r)}{3}$.

7. Simplify $\frac{5}{8}(2-3x)-\frac{3}{8}(1-2x)+\frac{7}{8}(4x-3)$.

8. Bring $\frac{7x-21}{14}$ to a denominator 2.

9. Multiply $7c+4d-2r$ by -5.

10. A and B are 40 miles apart. I do a portion of the journey at 10 miles an hour and the rest at 20 miles an hour and take 3 hrs. 15 min. altogether. What portion did I do at the slower rate?

EXERCISE 17 (*a*). MENTAL

Write down the answers without showing any working.

1. Solve the equation $\frac{4a}{7}=\frac{5}{21}$.

2. What is half the sum of $3x+2y$ and $5x+2y$?

3. If $5x+y=7$, find the value of y in an answer which includes an x term and a number.

4. How many times is $2a$ contained in $3b$?

5. Find the value of $\cdot 5x+\cdot 3y+\cdot 7z$ when $x=y=z=10$.

6. Simplify $-2(3-2a)-4(3b-2c)$.

7. Subtract the sum of $4a+3b$ and $2a+5b$ from $6a+10b$.

8. Bring $\frac{3q-4}{12}$ to a denominator 48.

9. What sum is 10 % more than £$10x$?

10. Write down the equation which will solve the following problem:

"A pole has $\frac{1}{6}$ of its length in mud, $\frac{1}{4}$ in water and 30 feet above water. What is its length?"

EXERCISE 17 (*b*).

1. Add together $3s - 4t + 2r$, $5t - 2r + 5s$, $2t + 3r + s$.

2. If q lbs. of sugar are bought for $5s.$, how many shillings will b lbs. cost?

3. Find the value of $3l + 5m$ when $l = 2{\cdot}5$, $m = 7{\cdot}5$.

4. Solve the equation $\frac{p}{2} + \frac{p}{3} + \frac{p}{4} + \frac{p}{6} = 20$.

5. Show by means of a diagram that $8 - (5 - 2) = 8 - 5 + 2$.

6. Simplify $5(2a - 3c) - 2(3a - 4c) - 6(a + c)$.

7. Find g when $\frac{5g - 4}{7} = \frac{3g - 2}{14} + \frac{1}{2}$.

8. By simplifying separately the two sides, prove that

$$\frac{2x - 7}{4} - \frac{x - 3}{5} = \frac{1 - 2x}{10} + \frac{x - 4}{2} + \frac{3}{4}.$$

9. Divide $35a + 14b - 21c$ by 7.

10. If I take a certain number of sixpences and 4 times as many pence, I get $8s.\ 4d.$ How many of each do I take?

SIMULTANEOUS EQUATIONS

Subtraction.

19. You have already learnt that $7-(5-9)=7-5+9=11$.

But $5-9=-4$, $\therefore 7-(-4)=11$.

This is merely another example of changing signs when brackets are removed.

Caution! Probably more mistakes are made through an incorrect treatment of the minus sign than from all other cases put together. As far as ordinary subtraction is concerned, the rule is that if you have to take a minus quantity from a plus quantity you really add two plus quantities.

Other results which should be noted are:

$$(-5)-(-2)=-5+2=-3,$$
$$(2)-(7)=-5,$$
$$(-6)-(9)=-15.$$

20. The following examples in vertical form include all types of subtraction:

From	7	3	4	-4	-6	-4	
Take	3	5	-2	-2	-9	5	
	4	-2	6	-2	3	-9	
i.e.	$7-3$	$3-5$	$4+2$	$-4+2$	$-6+9$	$-4-5$	......A,
	$=4$	$=-2$	$=6$	$=-2$	$=3$	$=9$	......B.

It may save you from many an error in subtraction if you always bring the subtraction mentally to the form shown in line A. From this, line B can quickly be obtained.

The same process is used in subtracting all like terms, for the result is always a like term to those used, and only the numbers have to be considered, e.g.:

From	$13a$	$14x$	$21y$	$-6x$
Take	$7a$	$19x$	$-3y$	$-3x$
Line A...	$(13-7)a$	$(14-19)x$	$(21+3)y$	$(-6+3)x$
Line B...	$6a$	$-5x$	$24y$	$-3x$

21. Caution! (*a*) *Unlike* terms cannot be subtracted in this way.

$$\begin{array}{ll} \text{If from} & 7x, \\ \text{you take} & \underline{8y,} \end{array}$$

you do *not* get $(7-8)x$, because unlike terms remain unlike both before and after subtraction. The right answer is $7x-8y$.

(*b*) Finally we have the cases:

$$\begin{array}{lll} \text{From} & 0 & 0 \\ \text{Take} & \underline{3x} & \underline{-3y.} \end{array}$$

The results are $0-3x$ or $-3x$; $0+3y$ or $3y$.

EXERCISE 18.

Horizontal Subtraction by Removing Brackets.

1. $(10)-(15)$.
2. $5-(-2)$.
3. $-7-(-4)$.
4. $2x-(5x)$.
5. $(2a+3b)-(a+b)$.
6. $(3x-7y)-(5x+8y)$.
7. $(3a+2b-c)-(2a-b-2c)$.
8. $(5x+7y-2z)-(3x-8y+2z)$.
9. $(115a+203b)-(108a-103b)$.
10. $85x+79y-(95x+109y)$.
11. $1-(3a-2b-c)$.
12. $0-(3x+5y-2z)$.
13. Take $3a-2b+5$ from zero.
14. Diminish $3x+7y-2z$ by $x-5y+8z$.
15. By how much is $24l+39m-17n$ greater than $19l-29m+16n$?
16. From a take $3a-2b+4c$.
17. Take $3x+7y-8z$ from $3x-8z$.
18. Take $2a-2b-7c$ from $2a-2b-7c$.
19. How much is $3a-2b-4c$ less than zero?
20. From $205x+307y$ take $(-233y)$.

EXERCISE 19.

Subtract the lower line from the upper in each case.

	1.	2.	3.	4.	5.
	7	7	5	-6	-3
	5	12	-2	-2	-7

	6.	7.	8.	9.	10.
	$2x$	$3y$	$4a$	$-3b$	$-2c$
	x	$4y$	$-2a$	$-6b$	c

	11.	12.	13.
	$x-y$	$3x+2y$	$2a+b$
	$2x+y$	$4x-3y$	$a-2b$

	14.	15.	16.
	$5x+7y$	$7x-5y$	$3c-4d$
	$3x-2y$	$9x+4y$	$2c-5d$

	17.	18.	19.
	$3x-2y$	$25x+17y$	$102a-29b$
	$5x-6y$	$35x-10y$	$56a+47b$

	20.	21.	22.
	$4x+5y$	$27a-45c$	$56y+47z$
	$-9y$	$-15a$	$66y-25z$

	23.	24.
	$3a-4b+c$	$5x-9y-2z$
	$2a+5b-2c$	$2x-4y+6z$

	25.	26.
	$8x+2y-3z$	$7a-3b+2c$
	$9x\qquad-4z$	$4a+5b$

	27.	28.
	$8x\qquad-2z$	$3a$
	$11x-5y+z$	$2a-2b+4c$

	29.	30.
	0	1
	$5x-2y+3z$	$3-2a+4b$

First simplify the parts of the following, then arrange in vertical form and subtract.

31. Take $2a - 5(b - c)$ from $5a - 3(2b + 5c)$.

32. Take $1 - 5(1 - x) + 2(1 - y)$ from $3 + 5x + 2y$.

33. From $3(5x - 2y) + 2(5x - 2z)$ take $2(x + 5y + z)$.

34. From $5(x - 3a + 2d)$ take $2(x - 2a + 4d)$.

35. If $4a - 7b$ is greater than $2(2a - 3b + 5)$, find the difference.

Simultaneous Equations.

22. Up to the present only equations with one unknown symbol have been introduced. We now come to equations with two unknown quantities, and the latter are usually, though not always, indicated by x and y.

Suppose $x + y = 7$.

This is true if $x = 1$ and $y = 6$
$x = 2$ $y = 5$
$x = 3$ $y = 4$
$x = 4$ $y = 3$
$x = 5$ $y = 2$
$x = 6$ $y = 1$ and so on.

Suppose we have another equation $x - y = 5$.

This is true if $x = 6$ and $y = 1$
$x = 7$ $y = 2$
$x = 8$ $y = 3$ and so on.

But however many pairs of values we may find for these equations taken separately, only one pair will be found to *satisfy* both equations. Here, for example $x = 6$, $y = 1$ satisfy both, and no others will do the work.

23. In this new type of equations you will always be given two equations in order to find a value for x and a corresponding value for y which will be true for *both* equations. But we shall not solve them by finding pairs of values as we did above.

Ex. 1. $$\left.\begin{aligned} 2x - y &= 9 \\ x - 2y &= 3 \end{aligned}\right\}.$$

Make the coefficients of x or y the same. Here take the x terms; to make them the same we multiply the bottom line by 2 and obtain $2x - 4y = 6$. Now we have

$$\left.\begin{aligned} 2x - y &= 9 \\ 2x - 4y &= 6 \end{aligned}\right\}.$$

Take the bottom line from the top.

$$(2x - 2x) - y + 4y = 9 - 6.$$
$$3y = 3,$$
$$y = 1.$$

Hence we are said to *eliminate* x.

Substitute this value of y in one of the given equations; *always choosing the simpler of the two.*

$$\begin{aligned} 2x - y &= 9. \\ \therefore 2x - 1 &= 9, \\ 2x &= 10, \\ x &= 5. \end{aligned}$$

$$\text{Hence } \left.\begin{aligned} x &= 5 \\ y &= 1 \end{aligned}\right\}.$$

Ex. 2. $\left.\begin{aligned} 3x + 2y &= 16 \\ 2x - y &= 6 \end{aligned}\right\}$. Eliminate y.

$\left.\begin{aligned} 4x - 2y &= 12 \\ \text{But } 3x + 2y &= 16 \end{aligned}\right\}$, i.e. multiply bottom line by 2.

Add: $7x = 28,$

$x = 4.$

Substitute in $3x + 2y = 16$:

$$\begin{aligned} 12 + 2y &= 16, \\ 2y &= 4, \\ y &= 2, \end{aligned}$$

$$\left.\begin{aligned} x &= 4 \\ y &= 2 \end{aligned}\right\}$$

Ex. 3. $$\left.\begin{aligned} 3x + 5y &= 27 \\ 5x - 4y &= 8 \end{aligned}\right\}.$$

Here it is necessary to multiply both equations in order to eliminate either x or y.

Rule for Elimination of x and y:

(1) Find the L.C.M. of the numerical coefficients of the symbol to be eliminated. Here it is 15 if we eliminate x.

(2) Multiply the two equations by such numbers as will produce $15x$. Here multiply the top line by 5 and the bottom by 3.

$$\left.\begin{aligned}15x + 25y &= 135\\ 15x - 12y &= 24\end{aligned}\right\}.$$

(3) If the symbols to be eliminated are both positive or both negative [and they are now, of course, equal], subtract. If the signs are different, add. Here we subtract.

$$15x - 15x + 25y + 12y = 135 - 24.$$
$$37y = 111,$$
$$y = 3.$$

(4) Having found one value, substitute this value in one of the original equations. Here take $3x + 5y = 27$.

$$3x + 15 = 27;\ 3x = 12;\ x = 4.\quad \left.\begin{aligned}x &= 4\\ y &= 3\end{aligned}\right\}.$$

It should scarcely be necessary to add that both equations must first be simplified until they include a single x term, a single y term, and a number, unless of course one of these is found to be missing altogether.

Thus $3x + y = 7 - 2y$ must first be changed to $3x + 3y = 7$.

$2x + 5y = 5x + 8y - 3$ to $-3x - 3y = -3$,

or better $3x + 3y = 3$,

or better still $x + y = 1$.

EXERCISE 20.

Solve the equations:

1. $\left.\begin{aligned}x + y &= 6\\ x - y &= 4\end{aligned}\right\}.$

2. $\left.\begin{aligned}x + 2y &= 8\\ x - 2y &= 4\end{aligned}\right\}.$

3. $\left.\begin{aligned}2x + 5y &= 22\\ 2x - 5y &= 2\end{aligned}\right\}.$

4. $\left.\begin{aligned}8x + 9y &= 25\\ 8x - 9y &= 7\end{aligned}\right\}.$

5. $\left.\begin{aligned}3x + 7y &= 17\\ x + 4y &= 9\end{aligned}\right\}.$

6. $\left.\begin{aligned}4x - 5y &= 26\\ 2x - 2y &= 12\end{aligned}\right\}.$

7. $\left.\begin{array}{l} 5x - 9y = 2 \\ 3x - 3y = 6 \end{array}\right\}.$

8. $\left.\begin{array}{l} 2x - 7y = 20 \\ 9x + y = 25 \end{array}\right\}.$

9. $\left.\begin{array}{l} 5x - 18y = 33 \\ 7x - 9y = 30 \end{array}\right\}.$

10. $\left.\begin{array}{l} 16x + 14y = 30 \\ 18x + 7y = 25 \end{array}\right\}.$

11. $\left.\begin{array}{l} 3x + 4y = 1 \\ 12x + 12y = 4 \end{array}\right\}.$

12. $\left.\begin{array}{l} 7x + 3y = 20 \\ 10x - 4y = 12 \end{array}\right\}.$

13. $\left.\begin{array}{l} 8x + 5y + 2 = 0 \\ 7x + 2y = 3 \end{array}\right\}.$

14. $\left.\begin{array}{l} 5x + 7y = 7 \\ 3y - 11x = 3 \end{array}\right\}.$

15. $\left.\begin{array}{l} 3x - 4y + 7 = 0 \\ 3y - 4x + 7 = 0 \end{array}\right\}.$

16. $\left.\begin{array}{l} 2x = 32 - 3y \\ 3x = 4y - 37 \end{array}\right\}.$

17. $\left.\begin{array}{l} 2x = y + 12 \\ x = 19 - 6y \end{array}\right\}.$

18. $\left.\begin{array}{l} 2x + 45 + y = 0 \\ x + y = 5(y - 9) \end{array}\right\}.$

19. $x + 3y = 4x + y = 11.$

20. $\left.\begin{array}{l} 4x - 13y = 191 \\ 3x - 10y = 93 \end{array}\right\}.$

21. $\left.\begin{array}{l} x = 1 - \dfrac{y}{2} \\ \dfrac{x}{2} + \dfrac{y}{3} = 1 \end{array}\right\}.$

22. $y - x = x + 11y = 10 + x + 6y.$

23. $\left.\begin{array}{l} 12x + 7y = 395 \\ 9x = 11y + 20 \end{array}\right\}.$

24. $\left.\begin{array}{l} 10y = 11x - 31 \\ 4y + 13 = 5x \end{array}\right\}.$

25. $\left.\begin{array}{l} 3x + 4y = 1 \\ 5x + 7y - 1 = 0 \end{array}\right\}.$

26. $\left.\begin{array}{l} x + 2y + 1 = \frac{2}{5}(x + 3y + 5) \\ 13 - 9y = 7x \end{array}\right\}.$

27. $3x + 2y = -(4x + 3y) = 1.$

28. $\left.\begin{array}{l} \dfrac{x}{2} + \dfrac{y}{3} = 20 \\ \dfrac{x}{3} + \dfrac{y}{2} = 20 \end{array}\right\}.$

29. $\left.\begin{array}{l} 3(x - 2y) + 4 = 10 \\ 3 - 2(3x - 4y) + 13 = 0 \end{array}\right\}.$

30. $\left.\begin{array}{l} \dfrac{3x + 2y}{3y - 4x} = 12 \\ x + y = 5 \end{array}\right\}.$

Problems involving Simultaneous Equations.

24. Ex. 1. Divide £15 between A and B so that B obtains £1. 10*s*. 0*d*. more than half of A's share.

$$\text{Let } A \text{ have } \pounds x,$$
$$B \quad\quad \pounds y.$$
$$\text{Then } x + y = 15 \text{ ....(1)}.$$
$$\text{But } B \text{ has } 1\tfrac{1}{2} + \frac{x}{2},$$
$$\therefore\ y = 1\tfrac{1}{2} + \frac{x}{2} \text{ ....(2)},$$
$$\text{or } 2y = 3 + x.$$
$$\left.\begin{aligned} 2y - x &= 3 \\ y + x &= 15 \end{aligned}\right\}.$$
$$\text{Adding } 3y = 18,\ y = 6,$$
$$\begin{aligned} 6 + x &= 15, \\ x &= 9. \end{aligned} \qquad \left.\begin{aligned} A \text{ has } &\pounds 9 \\ B \quad &\pounds 6 \end{aligned}\right\}.$$

Ex. 2. A number of two digits is such that the sum of the digits is 7. If the digits are reversed, the number is 9 less than before. Find the number.

$$\text{Let } x = \text{the ten's digit},$$
$$y = \text{unit's digit}.$$
$$\text{Then } 10x + y = \text{the number},$$
$$10y + x = \text{the number with digits reversed}.$$
$$\left.\begin{aligned} \therefore\ 10y + x &= 10x + y - 9 \\ \text{and } x + y &= 7 \end{aligned}\right\} \begin{aligned} &\text{.............................(1)}, \\ &\text{.............................(2)}. \end{aligned}$$

From (1), $9y - 9x = -9$.

$$\left.\begin{aligned} y - x &= -1 \\ \text{From (2),}\quad y + x &= 7 \end{aligned}\right\}.$$
$$2y = 6. \quad y = 3,\ x = 4.$$
$$\therefore \text{the number is } 43.$$

Ex. 3. A certain fraction is equal to $\frac{4}{5}$. If 2 is taken from numerator and denominator, the fraction becomes equal to $\frac{3}{4}$. Find the fraction.

$$\text{Let } \frac{x}{y} \text{ be the fraction.} \qquad \text{Then } \frac{x}{y} = \frac{4}{5} \text{ ....(1)},$$
$$\text{also } \frac{x-2}{y-2} = \frac{3}{4} \text{ ....(2)}.$$

$$\begin{aligned} \text{From (1)},\ 5x - 4y &= 0, & 20x - 16y &= 0 \\ (2),\ 4x - 8 &= 3y - 6, & 20x - 15y &= 10, \\ \text{or } 4x - 3y &= 2, & y &= 10. \\ & & 5x &= 4y. \\ & & x &= 8. \end{aligned}$$

$\therefore$ the fraction is $\frac{8}{10}$.

EXERCISE 21.

1. Find two numbers of which the sum is 17 and the difference 3.

2. A and B divide £17 between them and A has £4 more than B. How much has B?

3. A certain fraction becomes equal to unity if 2 is added to the numerator, and to $\frac{1}{3}$ if 4 is added to the denominator. Find the fraction.

4. The difference of two numbers is 12. One-sixth of the larger is 6 more than one-tenth of the smaller. Find the two numbers.

5. The perimeter of a rectangle measures 2 ft. 6 in., and one side is 5 in. longer than the other. Find the area.

6. In a purse there are 27 coins. Some of them are half-crowns and the others are shillings, and the total value is £2. 11*s*. How many of each are there?

7. Two candidates in an election poll 7500 votes. If 100 voters for the successful candidate had voted for the loser, the majority for the former would still have been 100. How many votes did the loser obtain?

8. I can row 10 miles an hour down stream and 4 miles an hour up stream. What is the rate of the current?

9. Find two numbers whose sum is 150 and which are such that twice the smaller exceeds the greater by six-sevenths of the smaller.

10. Three coins of one kind and four of another give a total value of £1. 5*s*. 0*d*. Four of the first and three of the second come to £1. 7*s*. 6*d*. What coins were used?

11. A certain fraction is equivalent to $\frac{1}{2}$, but if four is added to the numerator and four is taken from the denominator the result is unity. Find the fraction.

12. A man bought 264 eggs, some at three for $6d.$ and others at four for $6d.$ He paid £1. $19s.$ $0d.$ altogether. How many of each did he buy?

13. A certain number of scholars paid $2s.$ $6d.$ each for a school outing, and the parents who accompanied them paid $4s.$ $6d.$ each. If 250 went altogether and the total amount paid was £36. $5s.$, how many scholars went for the outing?

14. In a mixed department of 420 scholars there are 10 per cent. more girls than boys. How many boys are there?

15. Five years ago A was half as old again as B and the sum of their ages is six times the difference. Find A's age.

16. Six pounds of coffee and three pounds of tea cost $12s.$ Four pounds of tea and five pounds of coffee cost $13s.$ Find the cost of 1 lb. of each.

17. In a number of two digits the unit's digit is twice the ten's digit, and the value of the ten's digit is greater by 16 than that of the unit's digit. Find the number.

18. A, B and C have £600 among them. B has £30 more than A and C has £60 less than A and B together. How much has each?

19. A town is 36 miles from my home. I walk a certain number of miles to the station from my home and taxi twice as many miles at the other end. If the rail fare is $1\frac{1}{2}d.$ a mile and the taxi fare $1s.$ $6d.$ a mile, how many miles do I walk if the total expense is $9s.$ $9d.$?

20. Divide 810 into four parts such that the second is twice the first, the fourth twice the third, and the third 30 more than the first.

Transformation.

25. Sometimes it is necessary, or advisable, to find pairs of *integral*, i.e. whole number, values of x and y which satisfy a given equation in x and y.

Ex. 1. Find the integral values of x and y which will satisfy the equation: $$7x + 5y = 29.$$

(1) Transform so as to find an equivalent for either x or y. Here take x.

$$7x = 29 - 5y,$$

$$x = \frac{29 - 5y}{7}.$$

(2) We want to find integral values of x and y. We must here give such a value to y that $29 - 5y$ will divide exactly by 7.

If $y = 1$, we get $29 - 5$ (or 24)... not divisible by 7,
$y = 2$, we get $29 - 10$ (or 19)... not divisible,
$y = 3$, we get $29 - 15$ (or 14)... divisible, and $x = 2$.

After a little practice you will see that to get the next value for x you must take a value for y that is 7 greater or 7 less than before, 7 being the denominator of the fraction.

If $y = 10$ (7 more), we get $29 - 50$ or -21... divisible and $x = -3$, and

If $y = -4$ (7 less), we get $29 + 20$ or 49... divisible, and $x = 7$.

$\therefore$ three pairs are

x	7	2	-3,
y	-4	3	10.

Ex. 2. $4x - 5y + 1 = 0$.

Here we will transform so as to find an equivalent for y.

$$5y = 4x + 1,$$

$$y = \frac{4x + 1}{5}.$$

By trial, as above, we get $x = 1$, $y = 1$.

Hence we shall try $x = 6$ (5 more than 1); $x = 6$, $y = 5$.

Finally try $x = -4$ (5 less than 1); $x = -4$, $y = -3$ [or we might have taken $x = 11$ (5 more than 6); $x = 11$, $y = 9$].

$\therefore$ three pairs are

x	-4	1	6,
y	-3	1	5.

EXERCISE 22.

Transform the following equations in such a way that an equivalent is found for the symbols indicated:

1. $3y - 2x = 7$. (x)
2. $2a + 3b + 4 = 0$. (b)
3. $5 = 3x + 7z$. (z)
4. $7x = 2y + 8$. (x)
5. $0 = 7a - 2b + 5$. (a)
6. $6x - 5y = 11$. (x)
7. $7x + 5z = 0$. (z)
8. $5a + 4b + 2c = 0$. (a)
9. $\frac{x}{y} = 7$. (y)
10. $\frac{x}{2y} = 2$. (x)

11. $\frac{3x}{y} = \frac{1}{2}$. (x)

12. $\frac{2a}{3b} = \frac{1}{4}$. (b)

13. $\frac{x}{y} + 2 = 0$. (x)

14. $\frac{7x+2}{y} = 5$. (y)

15. $\frac{3x+2}{2y+5} = 7$. (x)

16. $\frac{4x+2y-3}{x+y-2} = 5$. (y)

17. $\frac{3x-2a+7}{7x+2a-3} = 1$. (x)

18. $\frac{x}{y} + 3 = \frac{2x}{y} - 1$. (y)

19. $\frac{x+1}{2y} = 3 - \frac{1}{y}$. (x)

20. $\frac{y}{2x+1} = \frac{3y-4}{x+\frac{1}{2}}$. (y)

EXERCISE 23.

Transform the following equations so as to give an equivalent for y, and then find the values of y when

$$x = -3, -2, -1, 0, 1, 2, 3.$$

1. $3x + 4y = 20$.
2. $7x - 3y = 15$.
3. $15x + 10y = 75$.
4. $5x + 9y = 43$.
5. $x + 8y = 7$.
6. $3x - 4y = 12$.
7. $5x = 2y + 11$.
8. $7 = 5x - 2y$.
9. $0 = 10x + 5y + 5$.
10. $7x = 40 - 5y$.

EXERCISE 24.

Transform the following equations so as to give an equivalent for *either x or y* and then find three pairs of integral values, where possible, of x and y which satisfy the equations; if impossible find any three pairs of values.

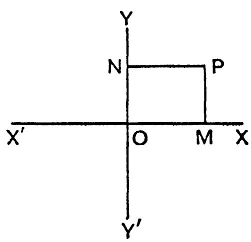
1. $5x + 3y = 16$.
2. $2x + 5y = 35$.
3. $6x - y = 7$.
4. $10x - 3y = 17$.
5. $4x = 3y - 9$.
6. $2x = 4y + 7$.
7. $0 = 5x - 2y + 18$.
8. $0 = 7x - 4y - 3$.
9. $4x - 2y = 2$.
10. $4y - 3x + 17 = 0$.
11. $2x + 3y = 24$.
12. $7y - 10x = 15$.
13. $23 = 2x + 5y$.
14. $x + 3y = 20$.
15. $4x = 5y - 5$.
16. $2 = 5x - 3y$.
17. $15 = 5x - 4y$.
18. $3x = 7 - 2y$.
19. $0 = 8 - 5x - 2y$.
20. $1 = 2x - 9y$.

Plotting Points.

26. You are probably acquainted with the method of fixing a point on the globe, by means of the latitude and longitude. The sailor, for example, knows exactly the latitude and longitude of the position in which the ship happens to be at any instant, and in case of distress a wireless message can be sent, which will be understood by all other mariners, giving the locality of the vessel.

Each line of latitude is parallel to the equator, and the distance from the equator is reckoned in degrees N. or S. of the equator. Each line of longitude goes from pole to pole, and each of these is a number of degrees, East or West of a "*meridian*" through Greenwich. Here we have two "*zero*" lines, one of latitude, the equator, and the other of longitude, through Greenwich; by reckoning the number of degrees, minutes and seconds of a place north or south of the first, and east or west of the second, we can easily fix the position of the place.

27. A similar device is adopted in mathematics to fix the position of a point on a page, two lines at right angles generally being used as zero lines.



These lines are often indicated, as shown in the diagram, by the letters X, X'; Y, Y'.

$X'OX$ is called the "*axis*" of X.

YOY' the *axis* of Y.

O the *origin*.

Let us suppose that we want to fix the position of the point P with respect to the "*axes*" of X and Y. If we know the distance of P from OY, we shall know that the point P lies somewhere on a line parallel to OY and the given distance away. If we know also the distance of P from OX, we shall know that P lies somewhere on a line parallel to OX and the given distance away.

But the observant pupil will see that two lines can be drawn parallel to OY and two more parallel to OX the given distances away. Hence P may be at any one of four points where these lines cross. This difficulty is removed by deciding that:

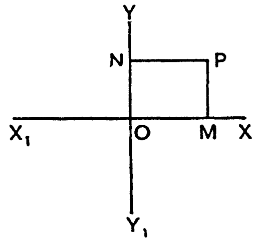
(1) Distances measured downwards from a point on to OX, or from OX upwards to the point are positive.

(2) Distances measured from right to left from a point on to OY, and those measured from OY to the right, are positive.

(3) Distances measured upwards from a point on to OX, or from OX downwards to the point, are negative.

(4) Distances measured from left to right from a point on to OY, and those measured from OY to the left are negative.

28. The following diagrams will make this clearer, PN and PM being the *coordinates* of the point P.

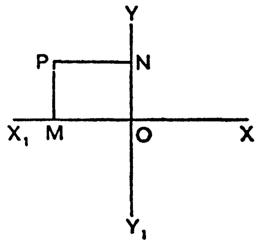


(1)

(1) Here PN and PM
or OM and ON } are positive.

If then PN is called x and PM y for the point P, both x and y for that point are positive.

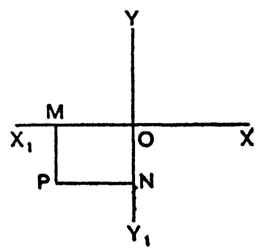
Note that in order to "*plot*" a point, we measure from O along OX (or OX_1) a distance equal to the x value for the point P, and then parallel to OY upwards or downwards a distance equal to the y value of the point P.



(2)

(2) Here we measure from O to M, i.e. in a negative direction, and then from M upwards to P, i.e. in a positive direction.

$\therefore$ for this point P, x is *negative*,
y is *positive*.



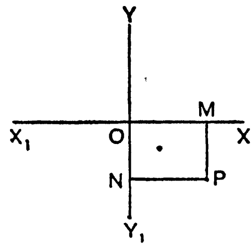
(3)

(3) Here we measure from O to M, i.e. negative, and from M downwards to P, i.e. negative.

$\therefore$ x is *negative*,
y is *negative*.

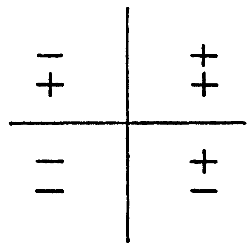
(4) Finally we have P in such a position that

x is *positive*,
y is *negative*.



(4)

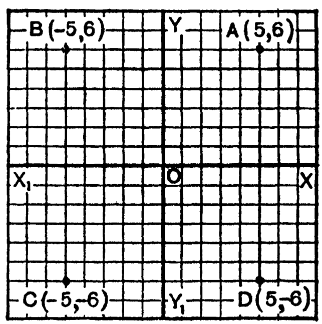
In diagram 5 will be seen in each "quadrant" the signs for x and y, the x signs being given first.



(5)

Diagram 6 shows four points A, B, C, D plotted, one in each quadrant.

A	is the point	$(5, 6)$,
B	,, ,,	$(-5, 6)$,
C	,, ,,	$(-5, -6)$,
D	,, ,,	$(5, -6)$.



(6)

Hence it should be clear that in order to plot a point we start at O and measure along OX if positive, or along OX_1 if negative, for the x value; then we measure at right angles to OX for the y value, upwards if positive, downwards if negative.

The unit of length chosen depends on the size of the paper supplied and the values of x and y given. Thus we may some-

times use $\frac{1}{10}''$ as unit for both x and y; or we may use $\frac{1}{2}''$, $1''$, and so on. Sometimes again we use different units for x and y. But always it is necessary to state what units are used for both. For example the units used in diagram 6 are $\frac{1}{10}''$ for x; $\frac{1}{10}''$ for y.

EXERCISE 25.

Using $\frac{1}{2}''$ as units for both x and y, plot, on one diagram for each question, the points A, B, C, D.

	A	B	C	D
1.	$5, \ 2$;	$-2, \ 3$;	$-5, -2$;	$4, -3$.
2.	$3, \ 1$;	$3, -4$;	$-2, \ 3$;	$-3, -2$.
3.	$-2, -1$;	$-3, -2$;	$1, \ 4$;	$1, -4$.
4.	$-2, \ 4$;	$-1, -1$;	$0, \ 2$;	$3, \ 0$.

5. When $3x+y=6$ we can find the following pairs of values of x and y which satisfy the equation:

x	3	2	1,
y	-3	0	3.

Using $\frac{1}{2}''$ as units for both x and y, plot the points and join them. You will find a straight line is formed.

6. Similarly $5x-3y=11$ gives the following table of values:

x	1	4	7	10	13,
y	-2	3	8	13	18.

Plot this straight line, using $\frac{1}{10}''$ as unit for x and y. You will find that all equations represented by $ax+by=c$, where a, b and c are any numbers, give straight lines.

For each of the following find *three* suitable pairs of values of x and y; plot the points and join them, using any suitable unit for both x and y.

7. $3x+y=9$.
8. $5x-7y=8$.
9. $4x+5y=0$.
10. $2x-5y+5=0$.
11. $3x+4y=1$.
12. $7x+3y=21$.
13. $3y-11x=3$.
14. $2x=32-3y$.
15. $x+y=5(y-9)$.
16. $13-9y=7x$.
17. Plot the points:

x	3	7	9,
y	4	4	4.

18. Plot the points:

x	-2	-2	-2,
y	5	3	-1.

Sometimes either the x or the y term in an equation is missing. In that case you can give the missing symbol any value you please, using with it the proper value of the other symbol.

Using this principle, draw the straight lines represented by the following equations:

19. $x=7$. 20. $2y+3=0$.

Graphical solution of equations.

29. Consider the equations:

$$\left.\begin{array}{ll}(1) & 2x+3y+1=0\\ (2) & 3x-2y-18=0\end{array}\right\}.$$

From (1), $y=\dfrac{-2x-1}{3}$,

x	-2	1	4,
y	1	-1	-3.

From (2), $y=\dfrac{3x-18}{2}$,

x	2	4	6,
y	-6	-3	0.

In plotting these points we should use the *largest possible scale.* For x we need to go from -2 to 6, i.e. 8 units; for y, from -6 to 1, i.e. 7 units.

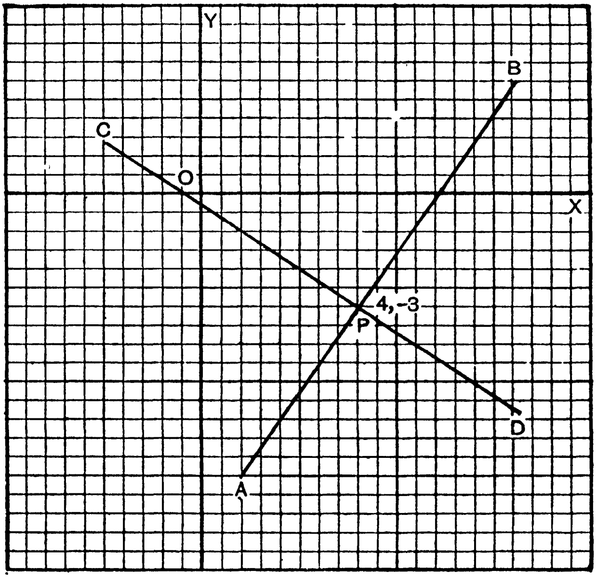
In the illustration these units are reduced for convenience to ·2″, ·2″.

The x axis will be near the top of the page, since most of the points lie farther below the line OX than above it.

The y axis will be towards the left of the centre of the page, because most of the points clearly lie to the right of OY.

Plotting the two sets of points and joining them, we get two straight lines AB, CD cutting at the point P, the coordinates of which are $4, -3$.

$$\therefore \left.\begin{array}{l}x=4,\\ y=-3\end{array}\right\}.$$



EXERCISE 26.

Solve the following equations graphically by drawing two straight lines, one for each equation of the pair, using separate axes for each question. *State in each case what units you use, and make them as large and convenient as you can.*

1. $\left.\begin{aligned} 3x + 2y &= 12 \\ 4x - y &= 5 \end{aligned}\right\}.$

2. $\left.\begin{aligned} 2x + 5y &= 22 \\ 2x + y &= 10 \end{aligned}\right\}.$

3. $\left.\begin{aligned} 3x + 2y &= 19 \\ 4x - y &= 18 \end{aligned}\right\}.$

4. $\left.\begin{aligned} 2x - 3y + 9 &= 0 \\ 3x - 4y + 11 &= 0 \end{aligned}\right\}.$

5. $\left.\begin{aligned} 4x - 5y &= 6 \\ 3x - 2y &= 1 \end{aligned}\right\}.$

6. $\left.\begin{aligned} 5x &= 4y + 31 \\ 2y &= 3x - 17 \end{aligned}\right\}.$

7. $\left.\begin{aligned} \frac{x}{3} - y &= 2 \\ \frac{x}{3} - \frac{2y}{3} &= 0 \end{aligned}\right\}.$

8. $\left.\begin{aligned} \frac{4x}{5} &= \frac{y-6}{5} \\ 3x - 4y &= 15 \end{aligned}\right\}.$

9. $\left.\begin{aligned} 2x + 3y + 6 &= 0 \\ 3x + 9y + 9 &= 0 \end{aligned}\right\}.$

10. $\left.\begin{aligned} \frac{3x}{4} - \frac{y}{6} &= 2\tfrac{1}{3} \\ 5x - y &= 24 \end{aligned}\right\}.$

11. $\left.\begin{aligned} 5x + 7y &= 24 \\ 5x &= 17 \end{aligned}\right\}.$

12. $\left.\begin{aligned} 3x - 2y &= 14 \\ 2y + 14 &= 0 \end{aligned}\right\}.$

SECTIONAL REVISION B

EXERCISE 27 (*a*). MENTAL

Write down the answers *without showing any working.*

1. Change $2x+5y=7$ so as to give the value of x in terms of y.

2. If $4x+3y=6$ and $4x-2y=1$, find y.

3. Perform the subtraction

$$\begin{array}{r} 2a-3l+k \\ a-5l-k \\ \hline \end{array}$$

4. If $5x-7y=19$ and $y=3$, find x.

5. Simplify $2(3a-2b)-5(2c+3d)$.

6. Reduce to lowest terms

$$\frac{4x}{8y+12m}.$$

7. Solve the equation $\dfrac{2}{x+1}=\dfrac{2}{2x}$.

8. If $\dfrac{3x}{y}=7$, find y in terms of x.

9. Find the cost of x apples at $7d.$ a dozen.

10. Write down the statements from which the following problem can be solved:

"Divide £35 into two parts in the ratio of 3 : 2."

EXERCISE 27 (*b*).

1. Solve the equations $\left.\begin{array}{r} 3x+4y=17 \\ 5x-3y=\ \ 9 \end{array}\right\}$.

2. Subtract $4s-3t-2r$ from $5s-t-5r$.

3. If $4x+7y=26$, obtain three pairs of integral values of x and y which will satisfy the equation.

4. Prove that $1-\dfrac{1-x}{3}+\dfrac{2-x}{4}=1+\dfrac{2+x}{12}$.

5. Solve the equation $\dfrac{3x-7}{5}=\dfrac{5x-3}{17}$.

6. If $2x - 3y + 7 = 4x - 5y + 8$, find x in terms of y.

7. Add together $4(3-2x)$, $3(2-3x)$, $5(1-2x)$.

8. Simplify $\dfrac{1-5x}{2} + \dfrac{3-7x}{4} - \dfrac{2x-4}{3}$.

9. A gross of apples is sold at $5d.$ a dozen, and another gross at $xd.$ a dozen. Find the average cost of an apple, and reduce your fractions to lowest terms.

10. Ten years ago one man was 5 years older than another, 25 years ago he was half as old again. How old are they now?

EXERCISE 28 (a). MENTAL

1. Change $y = \dfrac{3-x}{2}$ into the usual form with x and y terms on one side and the number on the other.

2. Solve the equations $x + y = 10$, $x - y = 4$.

3. Perform the subtraction

$$\begin{array}{r} 2c + 4m - n \\ c \phantom{{}+4m} - 2n \\ \hline \end{array}$$

4. Find the value of $\dfrac{4l-3m}{2l+5m}$ when $l = 2$, $m = 1$.

5. Simplify $\dfrac{2}{3x} + \dfrac{3}{2x}$.

6. Reduce to lowest terms $\dfrac{3a-9b}{12a+18c}$.

7. Solve the equation $\dfrac{5}{2x} = \dfrac{7}{2}$.

8. Add together $\cdot 5a$, $\cdot 7a$, $\cdot 9a$.

9. How many miles can a train go in x hours if it travels at 5 miles an hour?

10. Write down the equations which will solve the following problem:

"The numerator of a fraction is half as great again as the denominator and the sum of numerator and denominator is 10. Find the fraction."

EXERCISE 28 (*b*).

1. Find the sum of $3a - 1\frac{1}{2}c + \frac{1}{2}d$ and $2\frac{1}{2}c - \frac{1}{3}d - 3a$.

2. Solve the equations $x = \dfrac{4y-7}{3}$, $y = \dfrac{4x-7}{3}$.

3. Simplify $\dfrac{3(l-2m)}{8} + \dfrac{3l-5m}{12} - \dfrac{l-3m}{4}$.

4. Solve the equation $\dfrac{5}{x} + \dfrac{3}{2x} + \dfrac{7}{12x} = 170$.

5. Find the value of $-7(3a - 2b + 4c)$ when $a = b = c = -1$.

6. Subtract the sum of $4a - 6x$ and $2a - 6y$ from $12x - 12y$.

7. Reduce to lowest terms $\dfrac{5(2x-4y)}{2(5x-25y)}$.

8. Find three pairs of integral values that satisfy the equation
$$5x = 2y + 17.$$

9. If I can motor x miles in y hours, how far can I go in 5 hours?

10. Find two numbers whose difference is 8, such that half the larger and one-fifth of the smaller together equal 11.

EXERCISE 29 (*a*). MENTAL

1. If $5x + 7y = 29$, find y when $x = 3$.

2. Change $3x - 8y = 17$ to an equivalent equation containing $-16y$.

3. Subtract the bottom line from the top
$$\begin{array}{c} 1 \\ 4 - 7q + 5r \\ \hline \end{array}$$

4. Find the value of $\dfrac{2a+3b}{3a+2b}$ when $a = 1$ and $b = 1$.

5. Simplify $\dfrac{1}{a} + \dfrac{1}{2a}$.

6. Reduce to lowest terms $\dfrac{28a+7d}{14x+7y}$.

7. Solve the equation $2(x-1) = 8$.

8. Add together $\frac{1}{2}a + \frac{1}{3}b + \frac{1}{3}a + \frac{1}{2}b$.

9. What fraction has a numerator x and a denominator 5 less?

10. Write down an equation which will solve the following problem:

"A train travels 120 miles in a certain time. If half the journey was done at this rate, and the other half took only two-thirds as long, the whole journey would have taken $2\frac{1}{2}$ hours. How long did it take?"

EXERCISE 29 (*b*).

1. Add together $5a-(b-c)$, $2a-(b+2c)$, $4a-(b+3c)$.
2. Simplify $\dfrac{5x-7y}{4}-\dfrac{2x-5y}{3}+\dfrac{x-2y}{6}$.
3. Solve the equations $3a-7b=1$, $2a+7b=24$.
4. Find a table of integral values of x and y which will satisfy the equation $7x+3y=31$.
5. Solve the equation $\cdot 5\,(x-2)+\cdot 25\,(2x-3)=\cdot 75\,(x-4)$.
6. Subtract $l-3\,(2m-n)$ from $m-(3l-n)$.
7. By working out the two sides of the following equation, prove that $x=7$ satisfies it: $\dfrac{5x+1}{3}-\dfrac{2x+1}{5}=\dfrac{3x-7}{2}+2$.
8. If $7a+2b+1=3a-5b+2$, find b in terms of a.
9. What is the sum of three consecutive multiples of 5 of which the smallest is $x-2$?
10. Divide £70 among A, B and C so that B has only one-third as much as A and A eight times as much as C.

EXERCISE 30 (*a*). MENTAL

1. Reduce to a simpler form $x=\dfrac{3y-9}{4}$.
2. Solve for y: $\left.\begin{aligned}2x-5y&=9\\2x+3y&=17\end{aligned}\right\}$.
3. Take $2q-3r+5t$ from zero.
4. Find the value of $2a+7d$ when $a=7$ and $d=-1\frac{1}{2}$.
5. Simplify $-5\,(3-2x)+5\,(2-3y)$.

6. Reduce to lowest terms $\dfrac{16m+2n}{8m+n}$.

7. Bring $\dfrac{2m-3s}{15}$ to a denominator 30.

8. Take $2(a+b)$ from $3a+5b$.

9. Write down three consecutive even numbers of which $2x$ is the largest.

10. What equation will solve this problem?

"A is three times as old as B. In 6 years time he will be twice as old as B. How old are they now?"

EXERCISE 30 (*b*).

1. From $2(a-3b-2c)$ take $3(2b-4a+c)$.

2. Solve the equation $\dfrac{3s-1}{2}-\dfrac{2s-3}{5}=\dfrac{4s-7}{15}$.

3. Simplify $\dfrac{1-3x}{7}+\dfrac{5-2x}{14}-\dfrac{x-2}{2}$.

4. Add together $4(x+y)$, $3(2x-y)$ and $-2(y-3x)$.

5. Solve for r: $\left.\begin{aligned} 5r+2t&=29 \\ 3r-5t&=5 \end{aligned}\right\}$.

6. Prove that $x=5$ satifies the equation:

$$\frac{2x-3}{7}-\frac{5+2x}{5}=\frac{x-23}{9}.$$

7. Simplify $\dfrac{5}{2x}+\dfrac{3}{5x}-\dfrac{1}{6x}$.

8. Find three pairs of integral values which satisfy the equation $2x-5y=13$.

9. How far can I go in 6 hours at the rate of a miles in b hours?

10. I gave away half my money, then $\frac{1}{3}$ of the remainder and had £20 left. What had I at first?

INTRODUCTION TO QUADRATIC EQUATIONS

Multiplication.

30. Up to the present we have dealt mainly with very simple terms of one symbol, and expressions containing such terms, e.g. $4x$, $2a+3b$, $4a-l-5m$.

Usually even beginners in Algebra have some acquaintance with the formula for the area of a rectangle, viz. base $\times$ height, or bh, and of a circle, viz. πr^2, where $\pi =$ approximately $3\frac{1}{7}$. They may also know the formulae for the volume of a rectangular solid, viz. lbh (length $\times$ breadth $\times$ height); a cube, x^3; and a sphere, $\frac{4}{3}\pi r^3$.

We have now used three kinds of terms:

(1) Of the first dimension, i.e. using a simple symbol coupled with a constant quantity; e.g. $4a$, $3b$, $2\pi r$.

(2) Of the second dimension, i.e. using two symbols multiplied together, or one symbol multiplied by itself, coupled with a constant quantity; e.g. ab, $3x^2$, πr^2.

(3) Of the third dimension; e.g. x^3, abc, $\frac{4}{3}\pi r^3$.

These can be represented in geometry by lines, surfaces and solids, or by length, area and volume.

We can also have terms in algebra for which there is no simple equivalent in geometry, e.g. $4a^2bxy$, $x\sqrt[3]{y}$, c^3x^2yz. In all of them the presence of two or more symbols adjacent to one another indicates that they form a product, and the presence of an index above a symbol indicates that the symbol is to be multiplied by itself a number of times.

$$\text{E.g.}\quad x^2 = x \times x,$$
$$y^3 = y \times y \times y,$$
$$4ab^2x^3 = 4 \times a \times b \times b \times x \times x \times x.$$

Also $\sqrt{x}$ is a quantity which when multiplied by itself gives x, and $\sqrt[3]{x}$ one which when multiplied by itself and again by itself gives x.

The Index Law for Multiplication.

31. Since $x^2 = x \times x,$

and $x^3 = x \times x \times x,$

$$\therefore\ x^2 \times x^3 = x \times x \times x \times x \times x = x^5.$$

The rule is that when two "*powers*" of the same symbol are multiplied together, the indices are *added* in order to find the new index.

Only like terms can be collected together for addition and subtraction. But any two terms can be multiplied together, and the same rule is used for all: for each letter in the terms multiplied add together the indices for that letter in those terms.

$$\text{E.g. } ax^2 \times abx = (a \times a) \times b \times (x \times x \times x)$$
$$= a^2bx^3,$$

and $ax^2 \times bx \times b^2 xy = ab^3x^4y.$

Numbers are multiplied together as usual.

$$\text{E.g. } 3a^2 \times 2a = 6a^3,$$
$$4ax \times 3by = 12\,abxy.$$

Note that it is usual to place the letters in alphabetical order.

EXERCISE 31.

Multiply:

1. $3x$ by 4.
2. $2a$ by 5.
3. y by 6.
4. x by $2x$.
5. $3x^2$ by 4.
6. $3y$ by $-y$.
7. $2y^2$ by -4.
8. $2x^2$ by $-3x$.
9. $-2x$ by $-3x$.
10. $-5x^2$ by -1.
11. $-5x$ by $3x$.
12. $-7x$ by $-3x$.
13. x^2 by x.
14. $3x^2$ by $4x$.
15. $-2x^2$ by -4.
16. $-3y$ by y^2.
17. $4x^2$ by $-x$.
18. $2x$ by $3y$.
19. $-2x$ by $-3y$.
20. $-3a$ by $4b$.
21. $(-a)^2$.
22. $(-2x)^2$.
23. $(-3y)^2$.
24. $(-5x)^2$.
25. axy by bx.
26. c^2x by cx^2.
27. $-3abx$ by $-2ab^2$.
28. $-5x^3y$ by $5xy^2$.
29. $4ax^2$ by $-3bx^2$.
30. $-15x^3y$ by $-5x^2$.

32. In the following examples *each* term of the given expression is multiplied by the multiplier.

$$\text{E.g. } (4x + 2y) \times 3x = 12x^2 + 6xy,$$
$$(3x - 7a) \times -2a = -6ax + 14a^2,$$
$$(ax + by) \times xy = ax^2y + bxy^2.$$

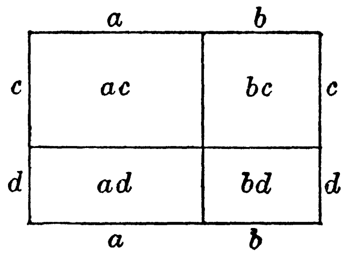
EXERCISE 32.

Multiply:

1. $2x+3y$ by 4.
2. $3x-2y$ by 5.
3. $5x-7y$ by 3.
4. $2a+3b$ by 6.
5. $2x-7y$ by -5.
6. $5y-3x$ by -4.
7. $-3x-5y$ by -4.
8. $-2a-9b$ by -8.
9. $-5x-8y$ by 3.
10. $-4b-9c$ by 7.
11. a^2+2a-7 by 4.
12. x^2+3x+9 by -5.
13. $2x^2-5x-8$ by -3.
14. $3x^2+9x-7$ by -6.
15. $3x-7$ by $-x^2$.
16. $4x+9$ by $-4x^2$.
17. $5x-3$ by $-3y$.
18. $2a+3b$ by $-2c$.
19. $7x+3y$ by $-2x$.
20. $5x-5y$ by $-5y$.
21. $ax+by$ by $-4ax$.
22. $3x^2+2xy$ by $-3bx$.
23. $3x^2+4x-2$ by $-2xy$.
24. $a^2+2ax+x^2$ by $-3ax$.

Product of $a+b$ and $c+d$.

33. We can represent $(a+b)(c+d)$ by a rectangle of which the sides are $a+b$ and $c+d$; and by drawing parallel lines we can split up the rectangle into four rectangles with sides a, c; a, d; b, c; b, d.



The diagram shows the rectangle $(a+b)(c+d)$ and the four parts ac, ad, bc, bd.

$$\therefore\ (a+b)(c+d)$$
$$=ac+ad+bc+bd.$$

Hence to multiply together two "*binomial*" expressions, i.e. expressions with two terms, we multiply *each* term of the first expression by *each* term of the second, and add the four results.

Similarly
$$(a+b)^2=(a+b)(a+b)$$
$$=a^2+ab+ab+b^2$$
$$=a^2+2ab+b^2$$
and
$$(a-b)^2=a^2-2ab+b^2.$$

Note. Hence the square of any binomial is the *sum* of the squares of the two terms with *twice* the product.

Ex. 1. $(3x + 2y)(x + y) = 3x^2 + 3xy + 2xy + 2y^2$ etc.

Ex. 2. $(7x - 3y)(3x - 2y) = 21x^2 - 14xy - 9xy + 6y^2$ etc.

Ex. 3. $(3x^2 + 4y^2)(x - 2y) = 3x^3 - 6x^2y + 4xy^2 - 8y^3$.

Ex. 4. $(2a - 3b)^2 = 4a^2 + 9b^2 - 12ab$.

Similarly, expressions other than binomials can be multiplied by inspection.

Ex. 5. $(a + b + c)(c + d) = ac + ad + bc + bd + c^2 + cd$, though here it would be better to say

$$ac + bc + c^2 + ad + bd + cd,$$

using the second expression as multiplier.

Simplifications.

34. Ex. 1. $(3x - 2y)(x + y) + (3x + 4y)(x - y)$

$$= 3x^2 + xy - 2y^2 + 3x^2 + xy - 4y^2$$
$$= 6x^2 + 2xy - 6y^2.$$

Note how the two terms in xy in line 2 are obtained.

Ex. 2. $(3x - 2y)(x + y) - (3x + 4y)(x - y)$.

Caution! It is not safe to attempt to remove brackets as well as to change signs in one operation. Hence the steps recommended are:

$$= 3x^2 + xy - 2y^2 - (3x^2 + xy - 4y^2)$$
$$= 3x^2 + xy - 2y^2 - 3x^2 - xy + 4y^2$$
$$= 2y^2.$$

Ex. 3. $(5x - y)^2 - 2(2x + 3y)^2$

$$= 25x^2 - 10xy + y^2 - 2(4x^2 + 12xy + 9y^2)$$
$$= 25x^2 - 10xy + y^2 - 8x^2 - 24xy - 18y^2$$
$$= 17x^2 - 34xy - 17y^2.$$

Ex. 4. $(2x + 3)^2(3x - 2)$

$$= (4x^2 + 12x + 9)(3x - 2)$$
$$= 12x^3 + 36x^2 + 27x - 8x^2 - 24x - 18$$
$$= 12x^3 + 28x^2 + 3x - 18.$$

35. Many students will prefer the following method of statement for expressions other than binomial, but its use for binomial expressions is quite unnecessary.

Ex. 1.

$$\begin{array}{l} 3x^2 + 4xy + y^2 \\ \underline{2x - 3y} \\ 6x^3 + 8x^2y + 2xy^2 \\ \qquad - 9x^2y - 12xy^2 - 3y^3 \\ \hline 6x^3 - x^2y - 10xy^2 - 3y^3. \end{array}$$

Ex. 2.

$$\begin{array}{l} 3x^2 - 2xy + 4y^2 \\ \underline{2x^2 + 3xy - 4y^2} \\ 6x^4 - 4x^3y + 8x^2y^2 \\ \qquad 9x^3y - 6x^2y^2 + 12xy^3 \\ \qquad\qquad - 12x^2y^2 + 8xy^3 - 16y^4 \\ \hline 6x^4 + 5x^3y - 10x^2y^2 + 20xy^3 - 16y^4. \end{array}$$

36. When the indices of the symbols increase or decrease in regular order, e.g. x^3, x^2, x, number; x^3, x^2y, xy^2, y^3; and so on, we can dispense with the letters for the multiplication process, and use what is called the method of "*detached coefficients.*"

Ex. 3. E.g. $(3x^3 + 4x^2 - 2x + 1)(x^2 - 2x + 3)$

$$\begin{array}{l} 3 + 4 - 2 + 1 \\ \underline{1 - 2 + 3} \\ 3 + 4 - 2 + 1 \\ \quad - 6 - 8 + 4 - 2 \\ \qquad\qquad 9 + 12 - 6 + 3 \\ \hline 3 - 2 - 1 + 17 - 8 + 3. \end{array}$$

We know the first term is $3x^5$ and hence the answer is

$$3x^5 - 2x^4 - x^3 + 17x^2 - 8x + 3.$$

Caution!

37. (1) This method must not be used unless *all the terms proceed regularly.* It will not do for $ax^2 + bxy + cy^2$.

(2) The terms must be arranged in *ascending or descending order.* Hence such an expression as $4x^3 + 3x - 2x^2 + 1$ must be changed to $4x^3 - 2x^2 + 3x + 1$ before converting to $4 - 2 + 3 + 1$.

(3) If a term is missing from the sequence, a 0 must be substituted,

e.g. $5x^4 - 2x^2 + x - 3$ will become

$$5 \cdot 0 - 2 + 1 - 3.$$

It is immaterial whether you use $+0$ or -0 in the course of the multiplication.

EXERCISE 33. Mental

1. $(x+3)(2x+3)$.
2. $(3x+4)(2x+1)$.
3. $(3x+2)(x-3)$.
4. $(2x-1)(3x+2)$.
5. $(4x-3)(3x-6)$.
6. $(2x-5)(x-4)$.
7. $(3-2x)(4-3x)$.
8. $(5-8x)(3x-2)$.
9. $(a+3b)(2a-b)$.
10. $(3a+2b)(3a-2b)$.
11. $(5x-7y)(7y-5x)$.
12. $(4x-2y)(2y-3x)$.
13. $(7x+9)(8-3x)$.
14. $(-3x-2)(-7x-4)$.
15. $(-5x-3)(-2x-9)$.
16. $(-4x-7)(7x-3)$.
17. $(5x+y)^2$.
18. $(4a+b)^2$.
19. $(7x-3)^2$.
20. $(8c+d)^2$.
21. $(1-3k)^2$.
22. $(3x-7y)^2$.
23. $(3l+7m)^2$.
24. $(2-7x)^2$.
25. $(8r-5)^2$.
26. $(7d-9)^2$.
27. $(-4-5x)^2$.
28. $(-3-7x)^2$.
29. $(3q-2r)^2$.
30. $(-4s+m)^2$.

EXERCISE 34.

Simplify:

1. $(a-2b)(a+3b)+(3a+4b)(a-3b)$.
2. $(5x-7y)(x+y)+(2x+3y)(x+3y)$.
3. $(4s-3t)(2s-t)+(2s+5t)(3t-4s)$.
4. $(2x-3y)^2+(x+2y)^2$.
5. $5x(x-y)-(3x+2y)^2$.
6. $(4x-3)(2x+1)-(2x-3)^2$.
7. $3(x-y)(x+2y)-2(x-3y)^2$.
8. $5x(2x-3)-2(x-2)^2+(x-3)^2$.
9. $45x(x+1)-15(x+2)(x-1)-(x-1)^2$.
10. $4ab+3a(7a-2b)-(5a+b)(a-2b)$.

Multiply:

11. $2x^2 - 7x + 4$ by $3x + 5$.
12. $4x^2 - 3x - 7$ by $x^2 + 5x - 2$.
13. $4x^3 + 2x - 1$ by $x - 1$.
14. $2a^2 - 3ab - 7b^2$ by $a - b$.
15. $(2a + 3b)(a + b)(2a - b)$.
16. $(x + 2)^3$. 17. $(3x + y)^3$. 18. $(4a - 3)^3$.
19. $(x + 1)(x + 2)^2$. 20. $(2x - 3)(3x - 4)^2$.

Division.

38. You have learnt that $x^3 \times x^4 = x^7$, i.e. we *add the indices for multiplication*. Conversely it is true that $x^7 \div x^4 = x^3$, i.e. we *subtract indices for division.*

Ex. 1. $(10x^4 - 8x^3 - 6x^2) \div 2x^2 = 5x^2 - 4x - 3$.

Ex. 2. $(16a^3b - 12a^2b^2 - 28ab^3) \div (-4ab) = -4a^2 + 3ab + 7b^2$.

When the terms to be divided, one by the other, contain more than one symbol, each symbol is dealt with separately:

$$\text{e.g.}\quad 5x^2y \div xy = (5 \div 1) \times (x^2 \div x) \times (y \div y)$$
$$= 5x.$$

$$12c^3d^2x^4 \div 4c^2dx^2 = 3cdx^2.$$

EXERCISE 35. MENTAL

Divide:

1. $3x^2$ by x. 2. $4x^2$ by $2x$. 3. $7a^2$ by $7a$.
4. $25x^2$ by 25. 5. $18a^2$ by 6. 6. $15c^2$ by $5c$.
7. $27b$ by -3. 8. $16b$ by -4. 9. $25a$ by -5.
10. $-18x^2$ by -3. 11. $-19y^2$ by -1. 12. $-24c^2$ by -6.
13. $-16a^2$ by $-4a$. 14. $-18c^2$ by $-3c$. 15. $-15x^2$ by $-5x$.
16. $-18y^2$ by 3. 17. $-14a^2$ by 7. 18. $-16x^2$ by 4.
19. $-x^4$ by x^2. 20. $-6y^4$ by $-y^3$. 21. $-15x^4$ by $5x$.
22. $-15x^4$ by $-5x$. 23. $-18y^4$ by $6y^3$. 24. $27x^4$ by $-9x^3$.
25. $3ab$ by a. 26. $4xy$ by $-2y$. 27. $6xy$ by $-3x$.
28. $4a^2b$ by $-2a$. 29. $3x^2y$ by $-3x$. 30. $-8y^2z$ by $-2y^2$.

EXERCISE 36. Mental

Divide:

1. $4x^3 + 3x^2 + 2x$ by x.
2. $3a^2 - 3ab$ by $3a$.
3. $5x^2 - 7x$ by $-x$.
4. $25x^3 - 15x^2y - 5x$ by $-5x$.
5. $4x^4 - 2x^3 + 3x^2$ by $-x^2$.
6. $-16x^4 + 14x^2 - 6x^3$ by $2x^2$.
7. $12x^2 - 16x - 8$ by -4.
8. $3a^3 - 4a^2b$ by $-a^2$.
9. $25x^4 + 15x^3 - 10x^2$ by $5x^2$.
10. $4tx^2 - 8cx^3 - 16x^4$ by $4x^2$.
11. $25x^3 - 20x^2y - 5xy^2$ by $-5x$.
12. $14x^3 + 21x^4 + 49x$ by $-7x$.
13. $x(x^2 - xy)$ by x^2.
14. $y^2(y^2 - 2y)$ by y^3.
15. $3x^2y - 6xy^2$ by $3xy$.
16. $4x^2y - 6xy^2$ by $-2xy$.
17. $4a^2 - 6ax$ by $-2a$.
18. $27x - 18x^2 - 24x^3$ by $-3x$.
19. $s^2(9 - 3s)$ by $3s^2$.
20. $m^2(9m - 3m^2)$ by $-3m^3$.

Finding Factors.

39. The method you have used of finding the product of two binomial expressions will help in obtaining the factors of an expression like $x^2 + 6x + 5$.

$$\begin{aligned}(x+1)(x+5) &= x^2 + 5x + x + 5\\ &= x^2 + \overline{5+1}\,x + 5.\end{aligned}$$

Let us work backwards from $x^2 + 6x + 5$:

x^2 is the product of x and x, the first terms of the binomial factors.

5 is the product of 1 and 5, the second terms of the factors.

6 is the sum of 1 and 5.

The factors are therefore $(x+1)(x+5)$.

The rule is, that if the first term of the given expression is x^2, y^2, a^2, etc., i.e. $1x^2$, $1y^2$, $1a^2$, the factors of the last term added together must give the coefficient of the middle symbol.

$$5 = 5 \times 1$$
$$6 = 5 + 1.$$

Ex. 1.
$$x^2 + 9x + 14.$$
$$14 = 7 \times 2.$$
$$9 = 7 + 2.$$
$$\therefore (x+7)(x+2).$$

Ex. 2. $x^2 + 15x + 36.$

Try $36 = 6 \times 6,$

15 does not equal $6 + 6.$

Try $36 = 4 \times 9,$

15 does not equal $4 + 9.$

Try $36 = 3 \times 12,$

$15 = 3 + 12.$

$\therefore (x + 3)(x + 12).$

Ex. 3. $x^2 - 7x + 12.$

Here the coefficient of x is -7.

Hence we have to find two factors of 12 which added together give -7. These are -3 and -4.

$\therefore (x - 3)(x - 4).$

Hence, if the last term of the expression is positive, the signs separating the two terms of the binomial factors are both the same. If the middle term is positive, they are both positive; if it is negative, they are both negative.

Ex. 4. $x^2 + 7x - 18.$

Here we have a minus sign for the third term. We have to find two factors of -18 which, added, give 7.

These are 9 and -2.

$\therefore (x + 9)(x - 2).$

Ex. 5. $x^2 - 3x - 180.$

Two factors of -180 which give -3 are -15 and 12.

$\therefore (x - 15)(x + 12).$

Or we may say: the factors of 180 which, subtracted, give 3 are 15 and 12, and *the minus sign must precede the larger number.*

EXERCISE 37.

Find the factors of:

1. $x^2 + 7x + 12.$
2. $x^2 + 11x + 30.$
3. $x^2 + 8x + 7.$
4. $x^2 + 9x + 8.$
5. $x^2 + 13x + 40.$
6. $x^2 + 12x + 20.$
7. $c^2 + 20c + 96.$
8. $a^2 + 12a + 27.$
9. $m^2 - 13m + 40.$
10. $t^2 - 18t + 77.$

11. $r^2 - 15r + 50.$
12. $q^2 - 12q + 32.$
13. $x^2 - 17x + 66.$
14. $c^2 - 26c + 160.$
15. $x^2 + 6x - 16.$
16. $x^2 - 2x - 35.$
17. $b^2 - 2b - 80.$
18. $x^2 + 5x - 84.$
19. $x^2 - 12x - 45.$
20. $x^2 + 14x - 32.$

Solution of Quadratic Equations.

40. We now come to a third type of equation, with only one symbol unknown, but with the *square* of that quantity introduced. Such an equation is known as *quadratic.*

$$\text{E.g. } x^2 - 5x + 6 = 0.$$

The factors of $x^2 - 5x + 6$ are $(x-2)(x-3)$.

$$\therefore (x-2)(x-3) = 0.$$

We want to find values of x (for there are always two in such equations) which will make the product of $x-2$ and $x-3$ equal to 0.

Obviously $x = 2$, or $x = 3$ will do this, for $2-2$ and $3-3$ are both 0.

If $x = 2$, then $(x-2)(x-3) = (2-2)(2-3) = 0 \times -1$; and any number of times 0 is 0. $x = 3$ gives the same result.

41. By and by you will get equations of which the expression will not factorise in this way, and another method must be introduced. But for the present all such expressions will factorise. The rule is therefore:

(1) Bring all terms to one side, and arrange so that the coefficient of x is positive.

(2) Find the factors of the expression.

(3) Put each factor equal to 0.

Ex. 1.
$$x^2 + 9x + 18 = 0.$$
$$(x+3)(x+6) = 0,$$
$$\begin{matrix} x+3=0 \\ x+6=0 \end{matrix} \qquad \left.\begin{matrix} x=-3 \\ x=-6 \end{matrix}\right\}.$$

Ex. 2.
$$x^2 + 2x - 35 = 0.$$
$$(x+7)(x-5) = 0,$$
$$\begin{matrix} x+7=0 \\ x-5=0 \end{matrix} \qquad \left.\begin{matrix} x=-7 \\ x=5 \end{matrix}\right\}.$$

Ex. 3.
$$x^2 - 4x + 4 = 0.$$
$$(x-2)(x-2) = 0,$$
$$\left.\begin{aligned} x-2=0 \qquad x=2 \\ x-2=0 \qquad x=2 \end{aligned}\right\}.$$

EXERCISE 38. MENTAL

Solve the equations:

1. $(x-4)(x-7) = 0.$
2. $(x-3)(x-8) = 0.$
3. $(x+6)(x+4) = 0.$
4. $(x+8)(x+9) = 0.$
5. $(x-2)(x+6) = 0.$
6. $(x-8)(x+5) = 0.$
7. $(x+4)(x+10) = 0.$
8. $(x+9)(x-7) = 0.$
9. $(x+8)(x+5) = 0.$
10. $(x+11)(x-8) = 0.$
11. $(x-55)(x+42) = 0.$
12. $(x+56)(x-87) = 0.$
13. $(x+29)(x+64) = 0.$
14. $(x+77)(x-33) = 0.$
15. $(x+108)(x-106) = 0.$
16. $(x+73)(x+63) = 0.$
17. $(x+2)^2 = 0.$
18. $(x+9)^2 = 0.$
19. $(x-11)^2 = 0.$
20. $(x-55)^2 = 0.$

EXERCISE 39.

Solve the equations:

1. $x^2 - 4x + 3 = 0.$
2. $x^2 - 6x + 8 = 0.$
3. $x^2 - 9x + 14 = 0.$
4. $x^2 - 8x + 15 = 0.$
5. $x^2 + 6x + 8 = 0.$
6. $x^2 + 10x + 25 = 0.$
7. $x^2 + 3x + 2 = 0.$
8. $x^2 + 5x + 6 = 0.$
9. $x^2 - 8x - 9 = 0.$
10. $x^2 - 7x - 30 = 0.$
11. $x^2 - 2x - 99 = 0.$
12. $x^2 - 3x - 28 = 0.$
13. $x^2 + 5x - 24 = 0.$
14. $x^2 + 5x - 36 = 0.$
15. $x^2 + 7x = -12.$
16. $x^2 + 8 = -9x.$
17. $40 = 13x - x^2.$
18. $x^2 + 160 = 26x.$
19. $80 = x^2 + 2x.$
20. $84 - 5x - x^2 = 0.$
21. $x^2 = 45 - 12x.$
22. $32 = x^2 + 14x.$
23. $\frac{1}{2}x^2 = 8 - 3x.$
24. $\frac{1}{5}x^2 + x = 16\frac{4}{5}.$
25. $3x^2 + 6x - 240 = 0.$
26. $4x^2 + 8x = 140.$
27. $\frac{1}{3}x^2 = 15 - 4x.$
28. $\frac{1}{6}x^2 + 6\frac{2}{3} = 2\frac{1}{6}x.$
29. $\dfrac{x^2}{5} = 3x - 10.$
30. $x^2 = 3(4x - 9).$

More Factors.

42. Very few non-fractional quadratic equations commence with x^2. Usually they are of the form $2x^2-3x-7=0$, or, expressing it more generally, $ax^2+bx+c=0$. Hence the method of factorisation of such an expression as $6x^2+x-12$ must be learnt.

Ex. 1. Consider the expression $(2x+5)(4x+3)$.

$$\begin{aligned}(2x+5)(4x+3)&=8x^2+6x+20x+15\\&=8x^2+\overline{6+20}\,x+15\\&=8x^2+26x+15.\end{aligned}$$

Suppose we have to factorise $8x^2+26x+15$.

For the first terms of the binomials we have $2x$, $4x$. But $8x^2$ might have been produced from x, $8x$.

For the second terms we have 5, 3. But 15 might have been produced from 3, 5; 1, 15; 15, 1.

The proper pairs can only be found by trial. The usual method is to put down the trial factors as follows:

Suppose we have $8x^2+26x+15$.

$$\text{Try } \begin{matrix}4 & & 5\\ & \times & \\ 2 & & 3\end{matrix} \quad \left[\text{i.e. } \begin{matrix}4x & & 5\\ & \times & \\ 2x & & 3\end{matrix}\right]$$

$$\begin{aligned}4\times 3&=12 \quad [\text{i.e. } 4x\times 3]\\ 5\times 2&=10 \quad [\text{i.e. } 2x\times 5].\end{aligned}$$

Look at the last sign of the expression. If this is plus, we have to *add* 12 and 10 to get the coefficient of x; if it is minus, we have to *subtract*. Here it is plus, and so we add. $10+12=22$, and not 26. Hence we must try again.

$$\text{Try } \begin{matrix}4 & & 3\\ & \times & \\ 2 & & 5\end{matrix} \quad \left[\text{i.e. } \begin{matrix}4x & & 3\\ & \times & \\ 2x & & 5\end{matrix}\right]$$

$$\begin{aligned}4\times 5&=20 \quad [\text{i.e. } 4x\times 5]\\ 2\times 3&=6 \quad\ \ [\text{i.e. } 2x\times 3].\end{aligned}$$

$20+6=26$. This is correct.

$\therefore (4x+3)(2x+5)$, since we know all signs are $+$.

Ex. 2. $6x^2-25x+14$.

$$\text{Try } \begin{matrix}3 & & 2\\ & \times & \\ 2 & & 7\end{matrix}$$

or 21, 4.

We *add* because the last term is +, and $21+4=25$. Therefore this arrangement is correct. But the middle sign is −, therefore $(3x-2)(2x-7)$.

Ex. 3. $6x^2-x-15$.

Here the last sign is −.

$$\begin{array}{ccc} 3 & & 5 \\ & \times & \\ 2 & & 3 \end{array}$$

$$9,\ 10.$$

We *subtract* the results and give the minus sign to the larger product, because the coefficient of x in the given expression is minus.

The larger product is 10, and this is produced from 2×5.

$$\therefore \begin{array}{ccc} 3 & & -5 \\ & \times & \\ 2 & & 3 \end{array}$$

or $(3x-5)(2x+3)$.

Ex. 4. $6x^2+x-15$.

$$\begin{array}{ccc} 3 & & 5 \\ & \times & \\ 2 & & -3 \end{array}$$

or $(3x+5)(2x-3)$.

Ex. 5. $15-x-6x^2$.

$$\begin{array}{ccc} 5 & & 3 \\ & \times & \\ 3 & & -2 \end{array}$$

or $(5+3x)(3-2x)$.

Caution! Always see if the x^2 or the number term comes first.

EXERCISE 40.

Find the factors of:

1. $6x^2+13x+6$.
2. $21x^2+73x+56$.
3. $15x^2+43x+8$.
4. $14x^2+53x+14$.
5. $12x^2-31x+20$.
6. $4x^2-32x+55$.
7. $35x^2-104x+77$.
8. $15x^2-46x+35$.
9. $2x^2-9x-35$.
10. $3x^2-4x-55$.
11. $25x^2+50x-11$.
12. $30x^2-101x-14$.
13. $25x^2-26x+1$.
14. $69x^2-32x+3$.
15. $180x^2-159x+35$.
16. $143x^2-290x+143$.

17. $180x^2 + 21x - 40.$
18. $125x^2 + 160x - 21.$
19. $24 - 61x + 35x^2.$
20. $35 - 52x + 12x^2.$
21. $21 - 166x - 75x^2.$
22. $125 + 90x - 8x^2.$
23. $25 - 70x + 49x^2.$
24. $49 - 154x + 121x^2.$
25. $16 - 104x + 169x^2.$
26. $81 - 252x + 196x^2.$
27. $9x^2 - 66x + 121.$
28. $16x^2 - 40x + 25.$
29. $144x^2 - 168x + 49.$
30. $225x^2 + 390x + 169.$

EXERCISE 41.

Solve the following equations, *putting each binomial factor separately equal to* 0:

1. $(3x - 4)(4x - 3) = 0.$
2. $(7x - 2)(8x - 3) = 0.$
3. $(4x - 11)(x - 5) = 0.$
4. $(x - 8)(7x - 3) = 0.$
5. $(9x - 5)(11x + 5) = 0.$
6. $(3x - 11)(7x + 11) = 0.$
7. $(45x + 7)(3x - 5) = 0.$
8. $(5x + 17)(3x - 7) = 0.$
9. $(8x - 3)(3x + 19) = 0.$
10. $(45x - 37)(35x - 83) = 0.$
11. $(215x - 7)(235x + 17) = 0.$
12. $(305x + 73)(403x - 85) = 0.$
13. $(75x + 11)(53x + 2) = 0.$
14. $(5x - 17)^2 = 0.$
15. $(7x - 15)^2 = 0.$
16. $(4x + 18)^2 = 0.$
17. $(3x - 2)(2x - 3)(3x - 4) = 0.$
18. $(3x - 17)(4x + 15)(2x + 11) = 0.$
19. $x(x - 7)(x - 8) = 0.$
20. $4x(x - 15)(3x + 73) = 0.$

Quadratic Equations involving Fractions.

43. Fractions will be dealt with in greater detail later. For the present it will be sufficient to deal with simple cases.

Ex. 1. $$\frac{3}{x} + \frac{2}{x - 1} = 3\tfrac{1}{2}.$$

The same rule is used as before, namely, multiply every term by the L.C.M. of the Denominators.

$$\text{Here L.C.M.} = 2x(x-1).$$
$$\therefore\ 6(x-1) + 4x = 7x(x-1),$$
$$6x - 6 + 4x = 7x^2 - 7x,$$
$$-7x^2 + 17x - 6 = 0,$$
$$7x^2 - 17x + 6 = 0,$$
$$(7x-3)(x-2) = 0,$$
$$x = \tfrac{3}{7} \text{ or } 2.$$

Ex. 2.
$$3x + \frac{5}{3x+2} = 18\tfrac{1}{4}.$$
$$\text{L.C.M.} = 4(3x+2).$$
$$\therefore\ 12x(3x+2) + 20 = 73(3x+2),$$
$$36x^2 + 24x + 20 = 219x + 146,$$
$$36x^2 - 195x - 126 = 0,$$
$$12x^2 - 65x - 42 = 0,$$
$$(x-6)(12x+7) = 0,$$
$$x = 6 \text{ or } -\tfrac{7}{12}.$$

Ex. 3.
$$\frac{7x+5}{x-2} = \frac{8x+2}{2x-5}.$$
$$\therefore\ (7x+5)(2x-5) = (8x+2)(x-2),$$
$$14x^2 - 25x - 25 = 8x^2 - 14x - 4,$$
$$6x^2 - 11x - 21 = 0,$$
$$(x-3)(6x+7) = 0,$$
$$x = 3 \text{ or } -\tfrac{7}{6}.$$

EXERCISE 42.

Solve the equations:

1. $2x^2 - 11x + 12 = 0.$
2. $4x^2 - 17x + 15 = 0.$
3. $3x^2 - 13x + 14 = 0.$
4. $2x^2 - 11x + 9 = 0.$
5. $8x^2 - 10x + 3 = 0.$
6. $21x^2 - 13x + 2 = 0.$
7. $8x^2 - 26x + 15 = 0.$
8. $3x^2 - 5x - 12 = 0.$
9. $10x^2 + 19x + 6 = 0.$
10. $14x^2 + 43x - 21 = 0.$
11. $15x^2 - 13x + 2 = 0.$
12. $7x^2 + 40x + 25 = 0.$
13. $12x^2 + 52x + 56 = 0.$
14. $2x^2 + x - 6 = 0.$
15. $4x^2 + 4x - 15 = 0.$
16. $55x^2 - 146x + 55 = 0.$

17. $27x^2 - 60x - 32 = 0.$

18. $15x^2 + 56x + 49 = 0$

19. $56x^2 + 37x - 55 = 0.$

20. $63x^2 - 62x + 15 = 0.$

21. $121x^2 = 154x - 49.$

22. $32x = 4x^2 + 55.$

23. $15x^2 = 46x - 35.$

24. $3x^2 = 4x + 55.$

25. $11 = 25x^2 + 50x.$

26. $26x = 25x^2 + 1.$

27. $180x^2 = 159x - 35.$

28. $24 = 61x - 35x^2.$

29. $21 = 166x + 75x^2.$

30. $104x - 16 = 169x^2.$

31. $\frac{3}{x-3} + \frac{3}{x-1} = 4.$

32. $\frac{1}{x} + x = 3\frac{1}{3}.$

33. $\frac{2}{x} + \frac{3}{x+2} = 3.$

34. $x + \frac{5}{x+1} = 5.$

35. $\frac{2}{x+3} + \frac{3}{x+2} = \frac{8}{15}.$

36. $\frac{1}{x} + \frac{5}{x-2} + 1\frac{1}{3} = 0.$

37. $\frac{60}{x+2} - \frac{60}{x+3} = \frac{2}{3}.$

38. $\frac{100}{x} = \frac{100}{x+1} + 5.$

39. $\frac{18}{x-3} = \frac{18}{x-4} - \frac{1}{4}.$

40. $\frac{24}{x} - \frac{24}{x+1} = \frac{1}{3}.$

41. $\frac{x+2}{2x+3} = \frac{3x}{3x+2}.$

42. $\frac{2x-3}{3x+5} = \frac{x-1}{5x+1}.$

Solution of Quadratics by "completing the square."

44. All the quadratics so far have been such that both the "answers" or "roots," i.e. the values of x, have been whole numbers or simple fractions. Not all equations can be solved so easily.

The method used below introduces the transformation of an equation such as $ax^2 + bx + c = 0$ in such a way that the equivalent of x is found.

Ex. 1. $3 = 7x + 6x^2.$

Step 1. Transpose the terms so as to obtain the x^2 term with a positive coefficient, followed by the x term, on the left, and the "number" term, called the "independent" term, on the right.

$$6x^2 + 7x = 3.$$

Step 2. Make the coefficient of x^2 unity. Here we divide throughout by 6.

$$x^2 + \tfrac{7}{6}x = \tfrac{3}{6}.$$

Step 3. Square *half* the coefficient of x and add to both sides; to the left in the form $(\frac{7}{12})^2$, and to the right in the form $\frac{49}{144}$.

$$x^2 + \tfrac{7}{6}x + (\tfrac{7}{12})^2 = \tfrac{3}{6} + \tfrac{49}{144}.$$

Step 4. Simplify the right-hand side only.

$$x^2 + \tfrac{7}{6}x + (\tfrac{7}{12})^2 = \frac{72+49}{144} = \frac{121}{144}.$$

Step 5. We now have the left side a "perfect square"; viz. of $x + \frac{7}{12}$.

Find the square roots of both sides, giving the right-hand side both positive and negative values because both $(\frac{11}{21})^2$ and $(-\frac{11}{21})^2$ give $+\frac{121}{144}$.

$$\text{I.e. } (x + \tfrac{7}{12})^2 = (\tfrac{11}{12})^2,$$

$$\therefore\ x + \tfrac{7}{12} = \pm\tfrac{11}{12}.$$

Step 6. Find x. $x = \frac{11}{12} - \frac{7}{12}$ or $-\frac{11}{12} - \frac{7}{12}$.

$$\text{I.e. } \frac{11-7}{12} \text{ or } \frac{-11-7}{12}$$

$$= \tfrac{4}{12} \text{ or } -\tfrac{18}{12}, \text{ i.e. } \tfrac{1}{3} \text{ or } -\tfrac{3}{2}.$$

Ex. 2. $3x^2 - 5x - 7 = 0$.

Step 1. $3x^2 - 5x = 7$.

2. $x^2 - \frac{5}{3}x = \frac{7}{3}$.

3. $x^2 - \frac{5}{3}x + (\frac{5}{6})^2 = \frac{7}{3} + \frac{25}{36}$.

4. $$= \frac{84+25}{36} = \frac{109}{36}.$$

5. $x - \frac{5}{6} = \pm\dfrac{\sqrt{109}}{6}$.

6. $x = \dfrac{5 \pm \sqrt{109}}{6}$.

In order to complete the answer we must now find $\sqrt{109}$ to as many places as are required, usually 2 or 3.

$$x = \frac{5 \pm 10{\cdot}44}{6} = \frac{15{\cdot}44}{6} \text{ or } \frac{-5{\cdot}44}{6}.$$

$$= 2{\cdot}57 \text{ or } -{\cdot}91 \text{ to 2 places.}$$

EXERCISE 43.

Solve the following equations by "*completing the square*," giving answers correct to two places of decimals where necessary.

1. $x^2 - 4x + 3 = 0$.	2. $x^2 + 12x + 32 = 0$.
3. $x^2 - 6x - 91 = 0$.	4. $x^2 + 8x - 128 = 0$.
5. $x^2 - 7x - 120 = 0$.	6. $x^2 + 3x - 88 = 0$.
7. $8x^2 - 10x + 3 = 0$.	8. $3x^2 - 5x - 12 = 0$.
9. $15x^2 - 46x + 35 = 0$.	10. $25x^2 - 26x + 1 = 0$.
11. $15x^2 - 16x - 15 = 0$.	12. $24x^2 - 7x - 6 = 0$.
13. $3x^2 - 8x - 3 = 0$.	14. $9x^2 - 9x = 10$.
15. $12x^2 - 7x - 12 = 0$.	16. $2x^2 - 13x + 15 = 0$.
17. $x^2 - 3x + 1 = 0$.	18. $x^2 - 6x + 2 = 0$.
19. $x^2 + x = 7x - 7$.	20. $1 - 4x + x^2 = 0$.

Solution of Quadratics by Formula.

45. By using the principle of completing the square we can obtain a formula for the solution of all quadratics, and this formula is more often used than the method of completing the square itself. In fact the student is advised not to use the latter method, but to use the formula when factors cannot be found easily.

Let $ax^2 + bx + c = 0$ represent the *general form* of the quadratic, where

The coefficient of x^2 is a,
" " x is b,
The independent term is c.

Step 1. $ax^2 + bx = -c$.

2. $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

3. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$.

4. $\qquad = \frac{-4ac + b^2}{4a^2}$ or $\frac{b^2 - 4ac}{4a^2}$.

Step 5. $x+\dfrac{b}{2a}=\dfrac{\pm\sqrt{b^2-4ac}}{2a}$.

6. $x=\dfrac{-\mathbf{b}\pm\sqrt{\mathbf{b^2-4ac}}}{\mathbf{2a}}$.

This formula must be learnt by heart.

46. Ex. 1. $3x^2-9x-8=0$.

Here $a=3, \quad 2a=6,$

$$\left.\begin{aligned} b&=-9, & -b&=9, & b^2&=81\\ c&=-8, & 4ac&=-96, & -4ac&=96\end{aligned}\right\}.$$

$$\therefore x=\frac{9\pm\sqrt{81+96}}{6}$$

$$=\frac{9\pm\sqrt{177}}{6}.$$

Ex. 2. $3x^2+3x-7=0$.

$a=3, \quad 2a=6,$

$$\left.\begin{aligned} b&=3, & -b&=-3, & b^2&=9\\ c&=-7, & 4ac&=-84, & -4ac&=84\end{aligned}\right\}.$$

$$\therefore x=\frac{-3\pm\sqrt{9+84}}{6}$$

$$=\frac{-3\pm\sqrt{93}}{6}.$$

Usually the answer is left in this form unless the question asks for decimal values.

EXERCISE 44.

Solve by formula, giving answers correct to two places of decimals where necessary. **Caution!** Bring all terms to the left side, and arrange so that the coefficient of x^2 is positive.

1. $3x^2-5x-2=0$.
2. $4x^2-2x=4$.
3. $7x^2-3x-4=0$.
4. $5x^2+14x=3$.
5. $(x-12)(x+3)=16$.
6. $(x-5)(x+3)=3(x-7)+20$.
7. $(x-3)(2x+1)=x^2-7$.
8. $(3+4x)(4-3x)=13$.
9. $2(2x-3)(x+2)-2(x-7)(x-2)=3(x-4)(x-1)$.
10. $x(x+3)(x+2)=(x-1)(x-2)(x+6)$.

11. $(3x+4)^2=(x-1)(x-22)$. 12. $(3x+1)(8x-5)=1$.

13. $(x-1)(x+2)+(x-2)(x+1)=(x+1)(x+2)$.

14. $x^2+(x-1)^2=4x(x-1)$. 15. $x(x-2)=x-1$.

16. $(5+x)(x-1)+(2x-5)(x-1)=4(5+x)$.

17. $x(x+1)=7(x-1)$. 18. $(1-x)(4-x)=3-x$.

19. $3(x+1)(x-3)+4(x+2)(x-3)+6(x+2)(x+1)=0$.

20. $(3x-2)(x-1)(2x+1)-(2x-3)(x-1)(2x+1)$
$=(2x+1)(x-1)(x-2)$.

Problems involving Quadratics.

47. Ex. 1. Divide the number 23 into two parts so that the square of one part together with twice the product of the two parts is equal to 465.

$$\begin{aligned}
\text{Let } x &= \text{one part,}\\
23-x &= \text{the other,}\\
2x(23-x) &= \text{twice the product,}\\
\therefore\ x^2+2x(23-x) &= 465,\\
x^2+46x-2x^2 &= 465,\\
-x^2+46x-465 &= 0,\\
x^2-46x+465 &= 0,\\
(x-15)(x-31) &= 0,\\
x &= 15 \text{ or } 31.
\end{aligned}$$

Obviously 31 cannot be one part of 23 and hence the answer is 15, 8.

Ex. 2. How much are eggs a dozen if two more for a shilling lowers the price a penny a dozen?

Let the price be x pence a dozen.

xd. for 12. $x-1$ pence for 12.

1*d*. for $\dfrac{12}{x}$. 1*d*. for $\dfrac{12}{x-1}$.

1*s*. for $\dfrac{144}{x}$. 1*s*. for $\dfrac{144}{x-1}$.

$$\frac{144}{x-1}-\frac{144}{x}=2.$$

$$144x-144(x-1)=2x(x-1).$$

$$144x-144x+144=2x^2-2x.$$

$$x^2-x-72=0. \qquad (x-9)(x+8)=0. \qquad x=9d.$$

EXERCISE 45.

1. One number exceeds another by 3 and the sum of their squares is 29. Find the two numbers.

2. The perimeter of a rectangle is 16 inches and the area is 15 square inches. Find the lengths of the sides.
(Note that the sum of two sides is 8 inches.)

3. The sum of two numbers is 18, and their product is 72. Find them.

4. The sum of the squares of two consecutive numbers is 481. Find the numbers.

5. Find two numbers which differ by 3, and of which the cubes differ by 117.

6. The square of a certain number is 13 more than 12 times the number. Find it.

7. The sum of a certain fraction and its reciprocal is $2\frac{1}{12}$. Find the fraction.

8. If A can travel 8 miles an hour faster than B, and B takes 2 hours longer than A to travel 60 miles, find their rates.

9. A train travels 120 miles at a uniform speed. If it had travelled 4 miles an hour slower, it would have taken 1 hour longer. Find its speed.

10. Find three consecutive numbers, the sum of the squares of which is 365.

11. If the speed of a train is reduced by 2 miles an hour, it takes 11 minutes more to travel 154 miles. Find its original speed.

12. A train would take 15 minutes longer over a run of 70 miles if it went 5 miles an hour slower. What is its speed?

13. Divide a line 15 inches long into two parts, so that the sum of the squares on the two parts equals 125 square inches.

14. Divide a line 10 inches long into two parts, so that the rectangle having these lengths as sides has an area of $18\frac{3}{4}$ square inches.

15. Last year the price of a certain kind of tea was x pence per lb., but now, the price having risen threepence per lb., x lbs. of tea cost 22*s*. 6*d*. Find x.

16. The length of a room is three feet greater than its breadth, and the area of the floor is 180 square feet. Find the length and breadth of the room.

17. The length of a room is twice as great as its breadth. If its length were increased by 4 feet and the breadth by 2 feet, its area would be increased by 104 square feet. Find the length and breadth.

18. A field consists of a rectangular plot 60 yards long and 45 yards wide. It is surrounded by a path of uniform width and of area 324 square yards. Find the width of the path.

19. A cyclist travels 39 miles at a uniform rate. If he had travelled 1 mile an hour faster, he would have taken 15 minutes less to do the distance. At what rate does he travel?

20. If the price of oranges were reduced a penny a dozen, I should get 10 more for 5 shillings than I do at present. What is the price of the oranges?

SECTIONAL REVISION C

EXERCISE 46 (*a*). MENTAL

1. Find the product of $(2a+3b)(3a-2b)$.
2. Divide $12x^2y-3xy^2+6y^3$ by $-3y$.
3. What are the factors of $l^2-5lm+6m^2$?
4. Solve the equation $(3x-5)(4x+3)=0$.
5. Show that $x=2$ satisfies the equation $2x^2-5x+2=0$.
6. If $4x-3y=0$, find x in terms of y.
7. Solve for s: $3s+2t=5$, $4s-2t=2$.
8. What is the coefficient of x in the product of
$$(2x-75)(x-1)?$$
9. If I can walk at the rate of m miles an hour, how long shall I take to walk n miles?
10. Write down the equation that will solve the following problem:

"If I motor at 30 miles an hour I can do a certain journey in 2 hours less than if I travel at 25 miles an hour. What is the length of the journey?"

EXERCISE 46 (*b*).

1. Simplify $(2x-3y)(x+2y)-(4x+3y)(x-2y)$.
2. Multiply $2x^2-x+2$ by $x-3$.
3. Solve the equation $\frac{1}{x}+\frac{1}{x-1}=\frac{9}{20}$.
4. Find the values of x when $y=-2, -1, 0, 1, 2$ if
$$2x+5y=21.$$
5. If $\frac{4x-3}{3x-2}=\frac{4x-5}{3x-3}$, find x.
6. Find the common factor of $5x^2-7x+2$ and x^2-7x+6.
7. Simplify $\frac{3}{x}+\frac{2}{3x}-\frac{1}{5x}$.
8. Simplify $\frac{4x^2+7x}{x}-\frac{2x^3-7x^2}{2x}+\frac{x^4-5x^3}{x^3}$.
9. Solve the equations $7x-5y=13$, $2x+3y=17$.

10. Two men A and B are 12 miles apart at noon and they are walking towards one another at 3 and 5 miles an hour respectively. At what time will they meet?

EXERCISE 47 (*a*). Mental

1. Multiply $2x^2 - 3x + 7$ by $-3x$.
2. Find the factors of $x^2 - 5x - 84$.
3. If $x - 3$ is one factor of $2x^2 + x - 21$, find the other.
4. If $3by = 4$, find y in terms of b.
5. Solve the equation $5(x-2) = 25$.
6. Remove the brackets from $(2s - 3t)(4s + 5t)$.
7. Add together $\frac{1}{x+2} + \frac{1}{x}$.
8. If $5x - 7y = 4x - 3y$, find y in terms of x.
9. How many times is $\frac{a}{x}$ contained in 1?
10. Write down an equation which will solve the problem: "Ten per cent. more than a given sum is £50 less than a sum half as much again. Find the given sum of money."

EXERCISE 47 (*b*).

1. Find the value of $\frac{(x+2y)(x-y)}{(2x+3y)(2x-y)}$ when $x = 4$, $y = 2$.
2. What are the factors of $15x^2 + 2ax - 8a^2$?
3. Solve the equation $\frac{1}{4}(x+3) + \frac{1}{3}(x-2) = \frac{1}{6}(2x-1)$.
4. Simplify $(5x-3)(2x-4) - (3x-1)^2 + (x+2)(x-3)$.
5. Solve the equation $\frac{1}{x} + \frac{3}{x+2} = \frac{3}{4}$.
6. Multiply $2x^2 - 5x + 3$ by $3x^2 + 2x - 3$, using detached coefficients.
7. Change $x+1$ for x in the equation $x^2 + x + 2 = 0$.
8. Simplify $\frac{x-2}{3} - \frac{x-3}{4} + \frac{x-4}{6}$.
9. If $\frac{3x-4}{y} = 2$, find x in terms of y.
10. A lawn has a length of 20 feet and a breadth of 15 feet. Find the width of the path all round it if the total area of lawn and path is 500 square feet, assuming that the path is of uniform width.

EXERCISE 48 (*a*). MENTAL

1. Simplify $(2a-4b)(a+b)$.
2. Solve the equation $6(x-3)=3x$.
3. If $2a+3b=22$, find a when $b=7$.
4. Divide $18a^3b^2-27a^2b^3$ by $-3a^2b^2$.
5. What is the sum of three consecutive numbers of which the smallest is y?
6. Find the value of $\sqrt{3x}$ if $x=\frac{3}{4}$.
7. Solve for y: $3x-7y=15$, $3x+2y=42$.
8. Simplify $\frac{2}{x}+\frac{5}{y}$.
9. If $3x+4$ is one factor of $6x^2-13x-28$, find the other.
10. Give an equation which will solve the problem: "Pears are $2d.$ a lb. more than apples, and I can get a lb. more of apples than of pears for $2s.$ Find the cost of each per lb."

EXERCISE 48 (*b*).

1. Simplify $\frac{2}{3(x-1)}+\frac{3}{2(x-1)}$.
2. Find a table of values for a and b if $3a-7b=0$.
3. Solve the equation $\frac{1}{5}(2x-1)+\frac{1}{3}(3x-2)=\frac{1}{10}(7x-5)$.
4. If $4x+7y+2(3x-2y)=\frac{1}{2}$, find y in terms of x.
5. Solve the equations $\frac{3}{x-2y}=\frac{4}{2x-3y}$; $x+2y=10$.
6. Find two values of x which satisfy the equation
$$\frac{2x+3}{3x-2}=\frac{5x+2}{5x}.$$
7. Simplify $3x(x^2-2x+1)-2(x-1)(x^2+x)$.
8. If $x=3$ satisfies the equation $ax+3=6x$, find the value of a.
9. Simplify $\frac{2a+b}{4}-\frac{a-3b}{5}$.
10. A boy does a journey in 4 hours on a bicycle. If he had travelled 2 miles an hour slower, he would have taken 5 hours. Find the length of the journey.

EXERCISE 49 (*a*). MENTAL

1. Simplify $3x(x-2y)-2x(3z-2)$.
2. Find the value of $\sqrt{2x+3y}$ when $x=y=5$.
3. Simplify $\dfrac{2}{x+1}+\dfrac{1}{x}$.
4. Find the factors of $r^2-15r+36$.
5. If $ax+7=14$ is satisfied by $x=2$, find the value of a.
6. Reduce to lowest terms $\dfrac{4x^2-28xy}{8x^2-12xy}$.
7. What is the coefficient of x^2 in the product of
$$2x^2+x-2 \text{ and } x^2-2?$$
8. If $y=x^2$, find a table of values for y when $x=-2, 0, 2$.
9. If I can row at x miles an hour, how far shall I go in y hours?
10. Write down two equations that will solve the following problem:

"If the numerator of a certain fraction is increased by 5 and the denominator by 2, the fraction will be increased by $\frac{3}{10}$. The sum of numerator and denominator is 17. Find the fraction."

EXERCISE 49 (*b*).

1. Simplify $(2a-3)^2-(a+3)(a-2)+(2a+1)(a-2)$.
2. Solve the equation $\frac{4}{5}(x-2)-\frac{3}{5}(3x+4)=2$.
3. Simplify $\dfrac{3}{2p-q}-\dfrac{2}{3p+q}$.
4. Find the factors of: $(x-2)(4x-6)+(2x+1)(2x-3)$.
5. Find two values of x which will satisfy the equation
$$\frac{2}{x}+\frac{2}{x+1}=\frac{11}{15}.$$
6. Make up an equation which has for its roots $\frac{2}{3}$ and $\frac{3}{4}$ and bring it to the form $Ax^2+Bx+C=0$.
7. Multiply $2x^2+5x-2$ by $2x^2-5x+2$.
8. If $4x+3y=3x-2y$, find y in terms of x.
9. Find the value of $5x^2-7x+6$ when $x=-5$.
10. A purse contains 2 more half crowns than ten shilling notes, and no other coins or notes. The total value is £2. 2*s*. 6*d*. How many notes are there?

GRAPHS OF STATISTICS AND GRAPHICAL PROBLEMS

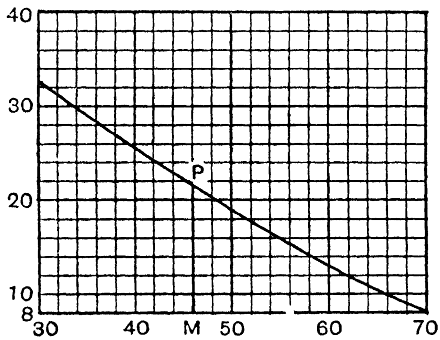
48. Ex. 1.

x	30	40	50	60	70
y	32·10	25·30	18·93	13·14	8·27

The x values represent ages and y the expectation of life for men at those ages. Plot the curve showing the relationship between x and y and find the expectation of life for a man 46 years of age.

Up to the present the axes for x and y have been considered as zero lines and have passed through the zero point 0.

It is clear that a very small scale will often need to be taken if this procedure is always adopted. In plotting statistics it is therefore usual to take axes starting at or near the lowest values of the connected quantities.



In the above problem our zero will be at 30 for x and at 8 for y.

Secondly, we are not always obliged to take the same scale for both x and y. Here, for example, x proceeds from 30 to 70; a difference of 40. It is clear therefore that 1″ can be the unit for 10, if we wish to make a large curve. y proceeds from about 8 to just over 30; a difference of about 22, and hence 2″ can be taken as the unit for 10. In the illustration given, however, the scale has been reduced for each, the units being

$$x \quad 10 = \tfrac{1}{2}'',$$
$$y \quad 10 = \tfrac{1}{2}''.$$

Needless to say we cannot plot accurately to two places of decimals on so small a scale.

The line PM indicates the expectation at 46, and this is found to be approximately 21·4.

Ex. 2. Construct a graph from which you can convert yards into metres and vice versa, given that 1 metre = 39·37″.

Take 10 metres = 393·7 inches
= 10·94 yards.

The scale depends entirely on the largest and smallest number of metres or yards it is desired to compare. Suppose 100 metres and 120 yards are the maximum limits. If the minimum limits are quite close to these, we can take a very large scale; if, however, we wish to take very small figures as well as large for comparison, a small scale must be chosen. In the diagrams given below we assume:

In Fig. 1 that 100 and 120 metres are the limits.

In Fig. 2 that 0 and 100 metres are the limits.

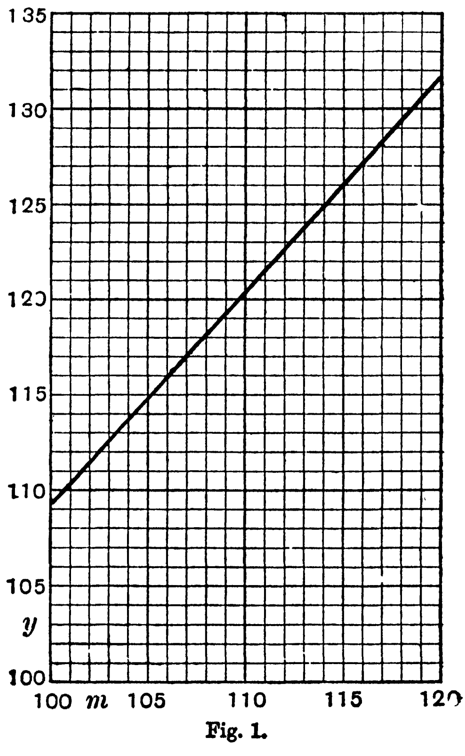


Fig. 1.

(*a*) Three points should be plotted. Take for example:

m	100	110	120
y	109·4	120·3	131·3

The scale shown in the diagram is 1″ for 10 in both cases.

Joining the three points we obtain a straight line, as we shall do in all cases where one quantity is *directly proportional* to another.

From the graph we can read off the number of yards corresponding to any number of metres.

E.g. 113 metres = approx. 123·6 yds.

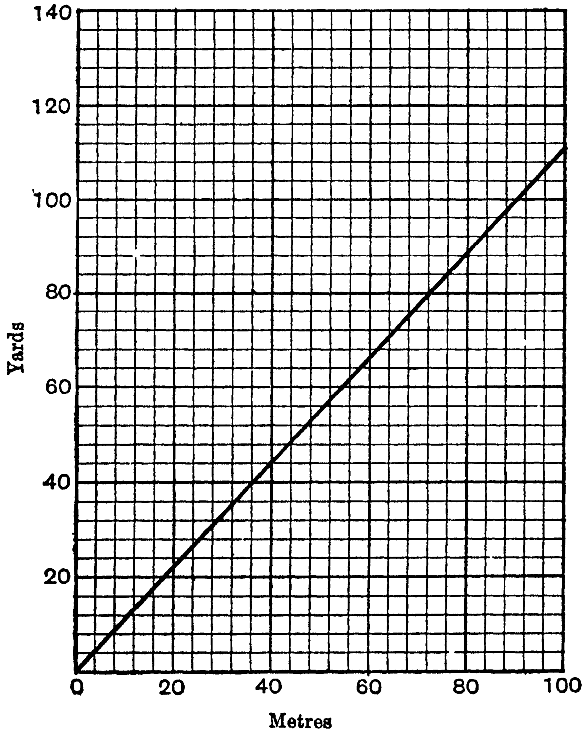


Fig. 2.

(*b*) Here we may take 1″ for 10 for both yards and metres. In the diagram a much smaller scale is taken. We know the graph is bound to be a straight line and hence two points are sufficient.

Caution! You should rarely be satisfied with two points, as a third forms a test of the other two, but here there is only one element of error since the straight line passes through the point 0, 0: 0 metres = 0 yards.

The results on this small scale are necessarily less accurate.

Ex. 3. *A* motors from a town *X* to another town *Y*, 175 miles away, stopping at each 50 miles for 12 minutes and travelling uniformly at 25 miles an hour. *B* starts from *Y* at the same time, travelling towards *X* and stops 24 minutes once only at the end of 50 miles, also at a uniform rate of 20 miles an hour. When will they meet?

A does 50 miles in 2 hrs. *AC* represents this.

He rests 12 mins. *CD* represents this.

There are 3 other similar sections, but two only need be drawn.

B starts from *Y* at 20 miles an hour, and rides towards *X*.

He does 50 miles in $2\frac{1}{2}$ hrs. *BF* represents this.

He rests 24 mins. *FG* represents this.

He does the next 50 miles in $2\frac{1}{2}$ hrs. *GH* represents this.

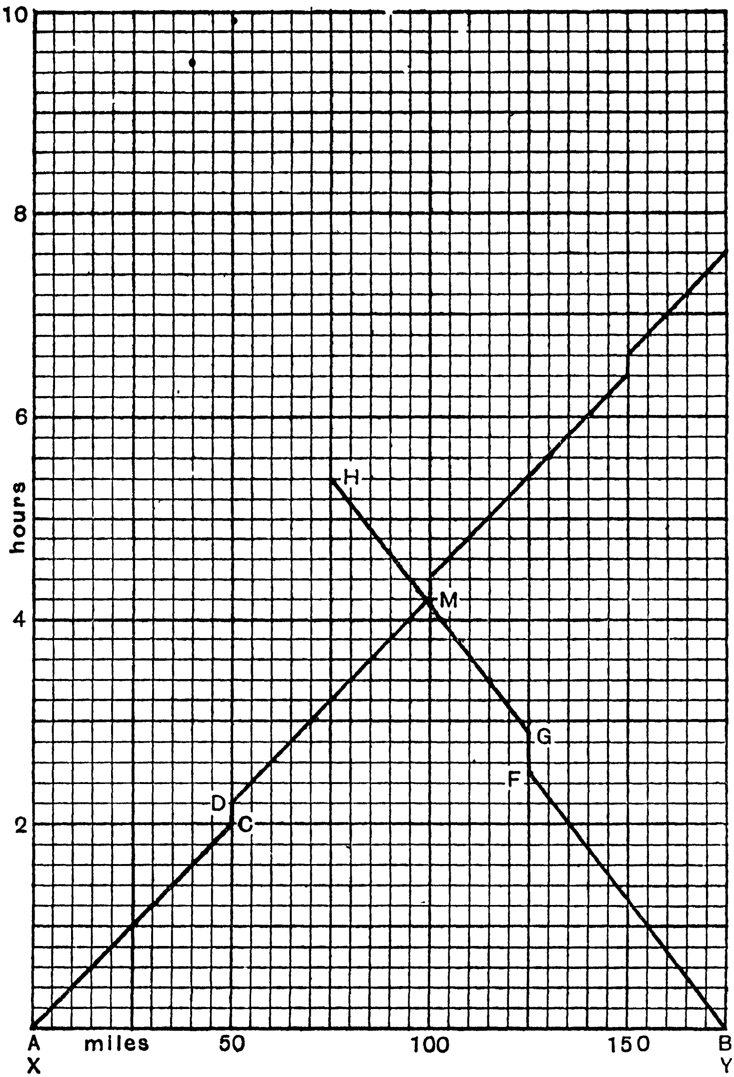
The lines cross at *M*, representing about 4 hrs. 12 mins. after the start.

[For diagram see page 85. 1″ = 50 miles; 1″ = 2 hours.]

EXERCISE 50.

1. 0° Centigrade = 32° Fahrenheit. 100° C. = 212° F. Draw a graph showing the relation between the Centigrade and Fahrenheit scales, and find the Centigrade reading corresponding to 73° F.

2. Draw a graph connecting kilometres and miles, using 1 kilometre = ·621 mile. From it find the number of kilometres in 3·35 miles, to the nearest first place of decimals.


10
8
6
4
2
hours
H
M
G
F
D
C
A
X
miles
50
100
150
B
Y

3. The marks obtained by a class range from 10 to 56. These are converted so that the range is from 40 to 100. If a boy received 39 marks on the first marking, how many will he receive on the second?

4. The area of a circle is given by πr^2, where r is the radius and π approximately 3·14. Draw a graph showing areas for circles of radii 1″, 2″, 4″, 6″, and from it read the area for a circle of 5″ radius.

5. The expenses of a school depend partly on the number of scholars and partly on permanent costs of maintenance. If the annual expenses for 200 boys is £2000 and for 400 is £3000, what will be the expenses for a school of 250 boys?

6. On a certain curve the x and y values of four points are:

x	1	3	5	10
y	13	19	25	40

Plot the points and join them, and from your graph find x when $y = 24$.

7. Find x when $y = 3·6$ if the following table gives the relation between x and y:

x	0	1	2	3	4	5	6
y	6·5	6·3	6·1	5·65	5	4	2·55

8. Plot the curve represented by

x	5·5	25	55·5	99·25
y	5	10	15	20

and from it find x when $y = 12$.

9. The depths of a river at various points across its breadth are given by the following table, x being the distance in feet from one shore and y feet the depth. Draw the section of the river.

x	0	7	12	18	25	32	40	50
y	4	8	10	15	18	14	10	5

10. The following table represents exports from a certain country from 1915 to 1920. Draw the graph showing the variation.

Years	1915	1916	1917	1918	1919	1920
Millions of £	75·4	76·0	82·3	95·7	102·8	120

11. The areas of cross section of a certain solid at points measured from one end are as follows:

Area in sq. ins.	314	176·6	78·5	19·6	0
Distance from end	0″	5″	10″	15″	20″

Find the area at a point 12 inches from the end.

12. A certain regular curve is plotted, of which the following are corresponding values of x and y:

x	4	4·2	4·4	4·7	5
y	5	4·8	4·65	4·35	4

Find y when $x = 4·6$.

13. Plot the following points and find y when $x = 25$:

x	1	5	10	20	40	50
y	20	12	11	10·5	10·25	10·2

14. In an experiment with a lever the following results were obtained in producing equilibrium:

Load in grams... ...	80	70	60	50	40	25
Distance of load from fulcrum in cm.	30	34·3	40·6	48·2	60	96

Illustrate these results by a graph on squared paper, and find from the graph what load had to be placed at 80 cm. from the fulcrum to produce equilibrium?

15. A feed pump of variable stroke driven by an electro-motor at constant speed gave the following experimental results:

Electrical horse-power	3·12	4·5	7·5	10·74
Power given to water	1·19	2·21	4·26	6·44

Plot on squared paper and state the probable electrical power when the power given to the water was 5.

16. The following table illustrates the changes in the volume of a gas when the pressure on it is increased. Draw a graph illustrating these results, and from it find out what the volume of the gas will be when the pressure is 50 inches of mercury,

and what the pressure will be when the volume of the gas is 8·5 cubic inches.

Pressure in inches of mercury	80	64	53·3	45·7	40	35·6	32	28·2
Volume in cubic inches	4	5	6	7	8	9	10	11

17. The following table gives the solubility of saltpetre in water at different temperatures, T being the temperature in degrees, and W the number of grams of the salt which dissolve in 100 grams of water. Plot a curve and say what the solubility of saltpetre is likely to be at 80° C.

T	0° C.	10°	20°	30°	40°	50°	60°	70°
W	12·5	19·5	29	43·5	61	83	108	131

18. A man starts to walk at $4\frac{1}{2}$ miles an hour. Half an hour later a second man starts from the same place and walks in the same direction at $5\frac{1}{2}$ miles an hour. How long after the second man starts will they meet, and how far will they be from the starting point?

19. Two men start cycling at the same moment, one from A in the direction of B 30 miles away, and the other from B in the direction of A. If the first rides at 10 miles an hour and the second at 12 miles an hour, where and when will they meet?

20. A motor omnibus leaves Bristol at 8.30 a.m. for Gloucester, 36 miles away, and travels uniformly at 12 miles an hour. It stops at Berkeley, 20 miles from Bristol, for fifteen minutes. Another bus leaves Gloucester for Bristol at 8.15 a.m. and also stops at Berkeley for fifteen minutes, travelling uniformly at 12 miles an hour. When and where will they meet?

21. A and B at a certain moment are 12 miles apart. A is walking towards B at 3 miles an hour and B is walking towards A at 5 miles an hour. Where will they meet?

22. A train which leaves a town A at 2.30 p.m. reaches another town B at 4.30 p.m. Another which leaves B at 2.50 p.m. reaches A at 4 p.m. If both trains travel at a uniform rate, find at what time they will pass.

23. A time-table for two trains, one a non-stop from A to D and the other stopping at B and C, is shown below. If the trains run uniformly, find when the non-stop passes the slow train.

A departs	9.50.	10.5.
B arr.	10.0.	
dep.	10.5.	
C arr.	10.20.	
dep.	10.30.	
D arr.	11.5.	10.50.

24. A cyclist riding from a town at 10 miles an hour passes the third milestone out of the town at 2 o'clock. A second cyclist riding in the same direction at 13 miles an hour passes the same milestone at 2.15. To which milestone will he be nearest when he overtakes the first cyclist?

Graphs of Equations.

49. You have already learnt that an equation of the type $ax + by + c = 0$ represents a straight line which can be plotted by obtaining a series of points. It is possible to solve equations of a higher degree in x or y than the first in a similar manner by giving values to x or y and finding corresponding values of y or x.

Ex. 1. $y = 3x^2 + 4$.

Let $x = 0$, then $x^2 = 0$, $3x^2 = 0$, $3x^2 + 4 = 4$;

$x = 1$, $x^2 = 1$, $3x^2 = 3$, $3x^2 + 4 = 7$;

$x = 2$, $x^2 = 4$, $3x^2 = 12$, $3x^2 + 4 = 16$;

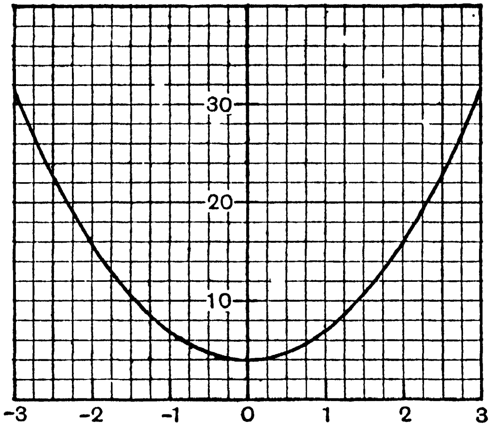
$x = -2$, $x^2 = 4$, $3x^2 = 12$, $3x^2 + 4 = 16$;

and so on.

These are generally set out as follows:

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$3x^2$	27	12	3	0	3	12	27
$3x^2 + 4$	31	16	7	4	7	16	31

Any convenient units can be used for x and y. In the diagram ·4″ has been used for x and ·05″ for y.



Plotting the points

-3	-2	-1	0	1	2	3
31	16	7	4	7	16	31

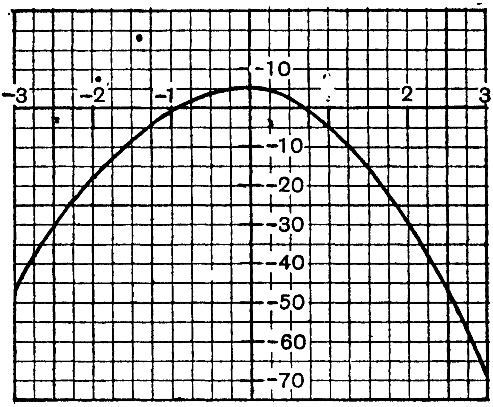
and joining them, we obtain a smooth curve of the type that will always be obtained from the quadratic equation.

Ex. 2. $y = 5 - 3x - 7x^2$ or $y = 5 - (3x + 7x^2)$.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$7x^2$	63	28	7	0	7	28	63
$3x$	-9	-6	-3	0	3	6	9
$3x + 7x^2$	54	22	4	0	10	34	72
$5 - (3x + 7x^2)$	-49	-17	1	5	-5	-29	-67

You will note that this curve has its turn at the *highest point*, whereas the curve for the first equation had its turn at the lowest point.

Note carefully too that the second curve *cuts the axis of* x, whereas the first does not meet the axis of x. This is an important point, since an equation such as $5x^2 + 7x - 3 = 0$ cannot be solved so as to obtain real answers unless the curve cuts the axis of x.



Ex. 3. Solve the equation $3x^2 - 14x = -8$.

The graphical method is as follows:

(1) Bring all terms to one side. $3x^2 - 14x + 8 = 0$.

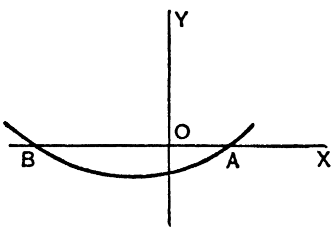
(2) Let y equal the expression so found. $y = 3x^2 - 14x + 8$.

(3) Find values for x and y and plot the curve.

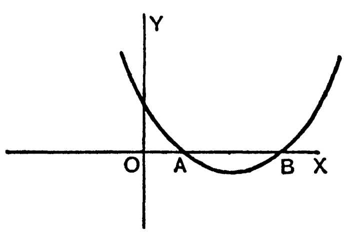
(4) See where the curve cuts the axis of x. If these points are called A and B it is clear that at A and B, and at no other points on the curve, $y = 0$; i.e. at A and B only, $3x^2 - 14x + 8 = 0$.

But OA and OB are the x values of these points. $\therefore$ where $x = OA$, $y = 0$ and where $x = OB$, $y = 0$. $\therefore$ OA and OB give the roots of the equation.

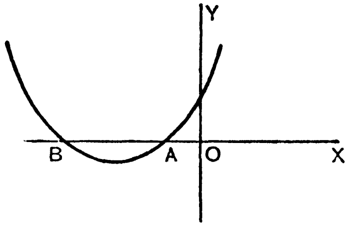
Some typical cases are shown in the diagrams below.



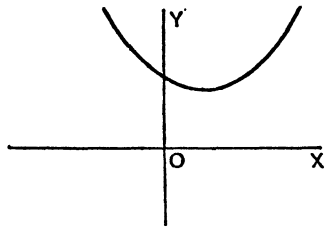
OA is $+$, OB is $-$.



OA is $+$, OB is $+$.



OA is –, OB is –.



There are no real roots.

EXERCISE 51.

Draw the graphs of:

1. $y = 3x^2$.
2. $y = 3x^2 + 4$.
3. $y = 4x^2 - 3x + 2$.
4. $y = 7x^2 + 5x - 3$.
5. $y = 3x^2 + 4x + 19$.
6. $y = 2x^2 + x + 23$.
7. $3y = 4x^2 + 2x - 1$.
8. $5y = 4x^2 + 3x - 2$.
9. $6y = \frac{1}{3}x^2 + 2x - \frac{1}{6}$.
10. $\frac{2}{3}y + 2 = \frac{3}{4}x^2 + 3x$.

Solve the equations, where possible. In cases where the curves do not meet the axis of x, state that there are no real solutions.

11. $2x^2 - 3x - 14 = 0$.
12. $x^2 - 7x + 6 = 0$.
13. $6x^2 - 17x - 80 = 0$.
14. $x^2 - 2x - 1 = 0$.
15. $2x^2 - 5x + 2 = 0$.
16. $x^2 - 5x - 1 = 0$.
17. $3x^2 + 4x - 9 = 0$.
18. $5x^2 = 7x + 5$.
19. $15x^2 - 7x = 3$.
20. $2x^2 + 3x + 15 = 0$.

LOGARITHMS

50. Consider the series of numbers:

$$10{,}000;\ 1000;\ 100;\ 10;\ 1;\ \tfrac{1}{10};\ \tfrac{1}{100};\ \tfrac{1}{1000}.$$

$$10{,}000 = 10^4;\ 1000 = 10^3;\ 100 = 10^2;\ 10 = 10^1.$$

Notice that the indices descend by 1: 4, 3, 2, 1.

This suggests a means of fixing indices for 1, $\frac{1}{10}$, $\frac{1}{100}$, etc., for the next indices after 4, 3, 2, 1 will be 0, – 1, – 2, – 3.

Hence it seems as if $1 = 10^0$; $\frac{1}{10} = 10^{-1}$; $\frac{1}{100} = 10^{-2}$; $\frac{1}{1000} = 10^{-3}$.

You have learnt that $x^5 \div x^3 = x^{5-3}$; $10^2 \div 10 = 10^{2-1}$, etc.

Hence $\dfrac{10}{10}$ or $\dfrac{10^1}{10^1} = 10^{1-1} = 10^0$. $\therefore 10^0 = 1$.

Again, $\dfrac{1}{10} = \dfrac{10^0}{10^1} = 10^{0-1} = 10^{-1}$.

Similarly $\dfrac{1}{100{,}000} = \dfrac{1}{10^5} = \dfrac{10^0}{10^5} = 10^{0-5} = 10^{-5}$, etc.

The series

10,000;	1000;	100;	10;	1;	·1;	·01;	·001;	etc.,
therefore corresponds to:								
10^4;	10^3;	10^2;	10^1;	10^0;	10^{-1};	10^{-2};	10^{-3};	etc.

Notice that, to the left of the line the index is one less than the number of digits in the number, and to the right of the line the numerical value of the index, which is always negative, is one more than the number of 0's after the decimal point.

Consider a number more than 100 but less than 1000, e.g. 200. The index will be more than 2 but less than 3. Mathematicians have discovered that the proper index is 2·3010 to 4 places; i.e.

$$10^{2\cdot3010} = 200.$$

Similarly $\qquad 10^{2\cdot4771} = 300$

and $\qquad 10^{2\cdot6857} = 485.$

Again consider a number more than 1 but less than 10, e.g. 7. The index will be more than 0 but less than 1: this is known by mathematicians to be about ·8451; i.e.

$$10^{\cdot8451} = 7.$$

Similarly $\qquad 10^{\cdot3010} = 2.$

$$10^{\cdot4771} = 3.$$

$$10^{\cdot6857} = 4\cdot85.$$

It will be noticed that we have used in the preceding paragraphs powers of 10 to represent numbers. When 10 is the *base*, the *index* required to produce a given number is called the *logarithm* of the number to the base 10, and although it would be possible to use any other number for a base, 10 is the one generally used. Tables have been constructed from which the required logarithm of a number can be read off.

The beginner always uses tables correct to four places of decimals only; in fact this degree of accuracy is sufficient for most practical purposes.

You are now ready to find the logarithms of whole numbers from the tables. The following extract from 4-figure tables will be used to illustrate the process.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7

Suppose the logarithm of 5286 is required. Split the number into 3 parts: 52, 8, 6.

(1) Look down the left-hand column for 52. This is the horizontal line you will use:

$$7160\ \ 7168\ \ 7177\ \ 7185\ \ 7193\ \ 7202\ \ 7210\ \ 7218\ \ \overset{8}{\boxed{7226}}\ \ 7235.$$

(2) Next consider the figure 8.

Look along your horizontal line until you see 8 above. This gives you 7226.

(3) Finally consider the last figure 6.

Look further along the same horizontal line; you will find the numbers

$$1\ \ 2\ \ 2\ \ 3\ \ 4\ \ \overset{6}{\boxed{5}}\ \ 6\ \ 7\ \ 7.$$

Keep looking until you see 6 at the top and note the number found. This is 5.

(4) Add 7226 and 5. This gives you 7231, which stands for ·7231.

(5) 5286 is more than 1000 but less than 10,000. The logarithm (or index) will be more than 3 but less than 4. Actually it is 3·7231.

$$\therefore\ 10^{3\cdot 7231} = 5286.$$

In practice these stages are performed quickly as follows: let us find *log* 5550 [short for the *logarithm of* 5550]. We write down 3, the whole number part of the log, and look along the 55 line till we see 5 above, and then add the decimal portion, arriving at 3·7443. The next figure is 0 and there is nothing more to add: if it were 9 we should add 7, and so on.

Test the following:

$$\begin{array}{ll} \log 5107 = 3\cdot 7082 & \therefore\ 10^{3\cdot 7082} = 5107. \\ \log 5318 = 3\cdot 7257 & \quad 10^{3\cdot 7257} = 5318. \\ \log 5499 = 3\cdot 7403 & \quad 10^{3\cdot 7403} = 5499. \end{array}$$

Now consider the log of 549·9.

$$\begin{aligned} 549\cdot 9 &= 5499 \div 10 \\ &= 10^{3\cdot 7403} \div 10^{1} \\ &= 10^{2\cdot 7403}. \end{aligned}$$

Hence we get the same decimal portion but a different whole number.

Thus we see how the following logs are obtained:

$$\begin{aligned} \log\ \ 51\cdot 07 &= 1\cdot 7082, \\ \log\ \ 5\cdot 318 &= \ \cdot 7257, \\ \log 549900 &= 5\cdot 7403, \end{aligned}$$

the whole number being always one *less* than the number of digits in the *integral* part of the given quantity.

Consider now the log of ·5499.

$$\begin{aligned} \cdot 5499 &= \ 5\cdot 499 \div 10 \\ &= 10^{\cdot 7403} \div 10^{1} \\ &= 10^{\cdot 7403 - 1} = 10^{-\cdot 2597}. \end{aligned}$$

Caution! The decimals given in the log tables are always positive. $\therefore$ $-$ ·2597 cannot be used as a log.

This introduces the first difficulty and we get over it as follows: $10^{\cdot 7403-1} = 10^{\bar{1}\cdot 7403}$. The minus sign is placed *over* the 1 to indicate

that it does not belong to ·7403, the latter being, as always, positive.

Again $$\cdot 005107 = 5\cdot 107 \div 1000$$
$$= 10^{\cdot 7082} \div 10^{3}$$
$$= 10^{\cdot 7082 - 3}$$
$$= 10^{\bar{3}\cdot 7082}.$$
$$\therefore \log \cdot 005107 = \bar{3}\cdot 7082.$$

For *purely decimal quantities* the whole number is always negative and one *more* than the number of 0's following the decimal point.

Caution! The whole number part of the log of 25·00107 = 1 and not $\bar{3}$, although there are two 0's after the decimal point. 25·00107 is *not a purely decimal quantity*.

Test the following results:

$$\log \cdot 05107 = \bar{2}\cdot 7082.$$
$$\log \cdot 5318 = \bar{1}\cdot 7257.$$
$$\log \cdot 00005499 = \bar{5}\cdot 7403.$$

Note that $\log 1 = 0\cdot 0000$, i.e. 0.
$\log 10 = 1\cdot 0000$, i.e. 1.
$\log 100 = 2\cdot 0000$, i.e. 2.

EXERCISE 52.

Find the logarithms to base 10 of the following numbers, using 4-figure tables:

1. 3367.	2. 4209.	3. 5039.
4. 2569.	5. 320·3.	6. 2·300.
7. 136.	8. 427.	9. 530.
10. 43.	11. 75.	12. 9·3.
13. 6.	14. ·7.	15. ·9.
16. ·3126.	17. ·4270.	18. ·5261.
19. ·2785.	20. ·3265.	21. ·9999.
22. ·0376.	23. ·00152.	24. ·000086.
25. ·00005.	26. ·0856.	27. ·003719.
28. ·0862.	29. ·00008536.	30. ·07.

Finding anti-logarithms.

51. For the reverse process we may either use the anti-logarithm tables or the body of the log table.

Ex. 1. Find anti-log $\bar{2}\cdot3196$, using log tables.

The line which contains the next smaller number than 3196 is:

	0	1	2	3	4	5	6	7	8	9
20	3010	3032	3054	3075	3096	3118	3139	3160	**3181**	3201

Hence the first three figures of our number are 208.

Subtracting 3181 from 3196 we get the *difference* 15. Using the same line as before we find

1	2	3	4	5	6	7	8	9
2	4	6	8	11	13	**15**	17	19

Hence the last figure is 7.

The significant figures of the anti-log are therefore 2087. To give $\bar{2}$ in the log we must have one '0'.

$\therefore$ Anti-log $\bar{2}\cdot3196 = \cdot02087$.

Ex. 2. Find anti-log 3·1034, using anti-log tables.

The line containing ·10 in the first column is:

	0	1	2	3	4	5	6	7	8	9
·10	1259	1262	1265	**1268**	1271	1274	1276	1279	1282	1285

Hence the anti-log of 3·103 is 1268.

Using the same line we find

1	2	3	4	5	6	7	8	9
0	1	1	**1**	1	2	2	2	3

Adding 1 we get 1269 as the anti-log of 3·1034.

Similarly anti-log $\bar{3}\cdot1034 = \cdot001269$.

EXERCISE 53.

Find the anti-logarithms of:

1. 1·256.	2. 2·084.	3. 1·0161.	4. $\bar{2}\cdot3614$.
5. $\bar{1}\cdot7802$.	6. $\bar{3}\cdot0016$.	7. $\bar{1}\cdot4500$.	8. $\bar{2}\cdot5784$.
9. 2·9187.	10. $\bar{3}\cdot0207$.	11. $\bar{1}\cdot5612$.	12. $\bar{3}\cdot7200$.
13. ·4726.	14. 2·0089.	15. ·1597.	16. $\bar{1}\cdot3096$.
17. ·8961.	18. $\bar{2}\cdot3609$.	19. $\bar{3}\cdot0018$.	20. ·0015.

Use of Logs for Multiplication.

52. Ex. 1. $37{\cdot}15 \times 6{\cdot}321 = 10^{1{\cdot}5700} \times 10^{{\cdot}8008}$
$$= 10^{1{\cdot}5700 + {\cdot}8008}$$
$$= 10^{2{\cdot}3708}.$$

Hence our answer is more than 100 but less than 1000.

This time we have to work backwards. The nearest decimal in the body of the page less than 3708 is 3692: we find 23 to the left of this and 4 above. Hence our first three figures in the answer are 234. To make 3692 into 3708 we add 16. Looking along the 3692 line we find 15 and 17 under 8 and 9 respectively. Hence our fourth figure in the answer is more than 8 and less than 9. From this we get our answer 2348 approximately to 4 figures.

Therefore $37{\cdot}15 \times 6{\cdot}321 = 234{\cdot}8$, since the answer is more than 100 and less than 1000, i.e. it has 3 digits in the whole number part.

Ex. 2. $4{\cdot}623 \times 1{\cdot}876$.

In practice we set out the sum as follows:

$$\log(4{\cdot}623 \times 1{\cdot}876) = {\cdot}6649 + {\cdot}2732$$
$$= {\cdot}9381$$
$$= \log 8{\cdot}671 \text{ or } 8{\cdot}672 \text{ or } 8{\cdot}673,$$

i.e. we cannot be sure to more than two places of decimals, and it will be safer to give an answer $8{\cdot}67$.

Ex. 3. ${\cdot}005619 \times {\cdot}5917$.

$$\text{Log}({\cdot}005619 \times {\cdot}5917) = \bar{3}{\cdot}7497 + \bar{1}{\cdot}7721$$
$$= \bar{3} + \bar{1} + 1{\cdot}5218$$
$$= \bar{3}{\cdot}5218$$

(In practice leave out the middle line.)

$$= \log {\cdot}003325.$$

Caution! Remember that $\bar{3}{\cdot}5218$ is a log. Hence we must look for it in the *body* of the page. It is only in *finding* a log that we start with the left-hand column, unless we use anti-logarithm tables.

EXERCISE 54.

Multiply together by using logs, and give answers to three significant figures:

1. $54{\cdot}62 \times 1{\cdot}609$. **2.** $325{\cdot}1 \times 1{\cdot}762$.

3. $29{\cdot}15 \times 15{\cdot}6$. **4.** $7{\cdot}108 \times 3{\cdot}290$.

5. $1{\cdot}069 \times 2{\cdot}156$.
6. $3{\cdot}276 \times 375{\cdot}2$.
7. $(1{\cdot}008)^2$.
8. $(32{\cdot}15)^2$.
9. $\cdot156 \times \cdot173$.
10. $\cdot142 \times 1{\cdot}62$.
11. $\cdot0863 \times \cdot08591$.
12. $\cdot00867 \times \cdot592$.
13. $1{\cdot}0596 \times \cdot008763$.
14. $2596 \times \cdot00000593$.
15. $\cdot00005876 \times \cdot0123$.
16. $\cdot005623 \times \cdot00003617$.
17. $(\cdot0156)^2$.
18. $(\cdot0019)^2$.
19. $\cdot326 \times 32{\cdot}6 \times 3{\cdot}26$.
20. $46{\cdot}1 \times \cdot0493 \times \cdot008624$.

Use of Logs for Division.

53. Ex. 1. $39{\cdot}7 \div 2{\cdot}106$.

$$\begin{aligned} \text{Log } (39{\cdot}7 \div 2{\cdot}106) &= 10^{1\cdot5988} \div 10^{\cdot3234} \\ &= 10^{1\cdot2754} \\ &= \log 18{\cdot}85. \\ \therefore\ 39{\cdot}7 \div 2{\cdot}106 &= 18{\cdot}85. \end{aligned}$$

Ex. 2. $\cdot3162 \div \cdot4179$.

$$\text{Log } (\cdot3162 \div \cdot4179) = \bar{1}\cdot5000 - \bar{1}\cdot6210.$$

Note that we must take $\cdot6210$ *from* $\cdot5000$.

The vertical form is perhaps better here:

$$\begin{array}{r} \bar{1}\cdot5000 \\ \underline{\bar{1}\cdot6210} \\ \bar{1}\cdot8790. \end{array}$$

This offers a difficulty. Having written down 8 we must add 1 to the $\bar{1}$ in the bottom line, obtaining 0. 0 from $\bar{1}$ is $\bar{1}$. $\therefore\ \bar{1}\cdot8790$.

$$\therefore\ \cdot3162 \div \cdot4179 = \cdot7567.$$

A few examples of this kind of subtraction will help the student.

$$\begin{array}{ccc} (1) & (2) & (3) \\ \bar{2}\cdot6471 & \bar{3}\cdot5496 & \bar{2}\cdot7802 \\ \underline{\bar{1}\cdot5972} & \underline{\bar{1}\cdot6932} & \underline{\bar{2}\cdot5806} \\ \bar{1}\cdot0499. & \bar{3}\cdot8564. & \cdot1996. \\ (4) & (5) & (6) \\ \bar{1}\cdot5932 & \bar{2}\cdot8762 & \bar{1}\cdot5086 \\ \underline{\bar{1}\cdot6408} & \underline{\bar{3}\cdot5876} & \underline{2\cdot8732} \\ \bar{1}\cdot9524. & 1\cdot2886. & \bar{4}\cdot6354. \end{array}$$

The operations with the whole number parts of the logs are as follows:

(1) $\bar{2}-\bar{1}=-2+1=-1.$

(2) $\bar{3}-(\bar{1}+1)=-3-0=-3.$

(3) $\bar{2}-\bar{2}=-2+2=0.$

(4) $\bar{1}-(\bar{1}+1)=-1+0=-1.$

(5) $\bar{2}-\bar{3}=-2+3=1.$

(6) $\bar{1}-(2+1)=-1-3=-4.$

EXERCISE 55.

Use logs to work the following divisions, giving answers to three significant figures:

1. 3964 ÷ 2498.
2. 427·3 ÷ 326·9.
3. 415000 ÷ 3976.
4. 5296000 ÷ 329500.
5. 27 ÷ 3·561.
6. 423 ÷ 3·108.
7. 5 ÷ 2·987.
8. 1 ÷ ·5672.
9. ·48 ÷ ·37.
10. ·2106 ÷ ·8596.
11. 1·562 ÷ ·003675.
12. 156·3 ÷ ·09876.
13. 1 ÷ ·05687.
14. 10 ÷ ·00398.
15. 1562 ÷ ·03987.
16. ·563 ÷ ·08791.
17. ·0564 ÷ ·0008591.
18. ·1589 ÷ ·0008359.
19. 156 ÷ ·0189.
20. ·1518 ÷ ·008937.

Use of Logs for finding Powers and Roots.

54. Ex. 1. Find the value of $(1{\cdot}05)^5$.

Since $(1{\cdot}05)^5 = 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05$, it is merely necessary to find 5 times the log of 1·05.

$$\begin{aligned}\text{Log } (1{\cdot}05)^5 &= 5 \log 1{\cdot}05\\ &= 5\,({\cdot}0212)\\ &= {\cdot}1060\\ &= \log 1{\cdot}276.\end{aligned}$$

$$\therefore (1{\cdot}05)^5 = 1{\cdot}276.$$

Ex. 2. Find the cube root of 7·089.

Now $7{\cdot}089 = x^3$.

Therefore three times the log of the cube root equals the log of 7·089, or the log of the cube root $= \frac{1}{3} \log 7{\cdot}089$

$$= \tfrac{1}{3}({\cdot}8506)$$
$$= {\cdot}2835$$
$$= \log 1{\cdot}921.$$
$$\therefore \sqrt[3]{7{\cdot}089} = 1{\cdot}921.$$

Ex. 3. Find $\sqrt{{\cdot}08962}$.

$$\text{Log} \sqrt{{\cdot}08962} = \tfrac{1}{2} \log {\cdot}08962$$
$$= \tfrac{1}{2}(\bar{2}{\cdot}9524)$$
$$= \bar{1}{\cdot}4762$$
$$= \log {\cdot}2994.$$
$$\therefore \sqrt{{\cdot}08962} = {\cdot}2994.$$

Ex. 4. Find $\sqrt{{\cdot}008962}$.

$$\text{Log} \sqrt{{\cdot}008962} = \tfrac{1}{2} \log {\cdot}008962$$
$$= \tfrac{1}{2}(\bar{3}{\cdot}9524).$$

Caution! We cannot say 2 into $\bar{3}$ is $\bar{1}$ and 1 over. It is -1 over, and this will not join with ·9524 since it is always positive. We must divide the minus whole number and the positive decimal separately. The difficulty is obviated by changing $\bar{3}{\cdot}9524$ into $\bar{4} + 1{\cdot}9524$.

Proceeding we get $\log \sqrt{{\cdot}008962} = \frac{1}{2}(\bar{4} + 1{\cdot}9524)$

$$= \bar{2}{\cdot}9762$$
$$= \log {\cdot}09466.$$
$$\therefore \sqrt{{\cdot}008962} = {\cdot}0947.$$

Ex. 5. $({\cdot}1623)^{\frac{5}{3}}$.

$$\text{Log} ({\cdot}1623)^{\frac{5}{3}} = \tfrac{5}{3} \log {\cdot}1623$$
$$= \tfrac{5}{3}(\bar{1}{\cdot}2103)$$
$$= \tfrac{1}{3}(\bar{4}{\cdot}0515)$$

[Note: $5 \times 2 = 10$. Carry 1. $5 \times \bar{1} = \bar{5}$. $\bar{5} + 1 = \bar{4}$.]

$$= \tfrac{1}{3}(\bar{6} + 2{\cdot}0515)$$
$$= \bar{2}{\cdot}6838$$
$$= \log {\cdot}04829$$
$$\therefore ({\cdot}1623)^{\frac{5}{3}} = {\cdot}04829.$$

Ex. 6. $(\cdot 1623)^{\frac{5}{3}} \div \sqrt{0\cdot 861}$. See Ex. 5.

$$\begin{aligned} \text{Log} &= \bar{2}\cdot 6838 - \bar{1}\cdot 7870 \\ &= \bar{2}\cdot 8968 \\ &= \log \cdot 07885 \text{ or } \cdot 07886. \end{aligned}$$

∴ Answer is ·0789 to 4 places, or to 3 *significant* figures.

Ex. 7. $\dfrac{\cdot 08962}{\sqrt[5]{\cdot 0861}} \times \sqrt[3]{(\cdot 1623)^5}$. See Ex. 6.

$$\begin{aligned} \text{Log} &= \bar{2}\cdot 9524 - \bar{1}\cdot 7870 + \bar{2}\cdot 6838 \\ &= \bar{3}\cdot 6362 - \bar{1}\cdot 7870 \\ &= \bar{3}\cdot 8492 \\ &= \log \cdot 007066 \text{ or } \cdot 007067 \end{aligned}$$

∴ Answer is ·00707 to 5 places, or to 3 *significant* figures.

EXERCISE 56.

Find the values of the following to 3 significant figures, using logs:

1. $(1\cdot 567)^3$.
2. $(1\cdot 008)^7$.
3. $(\cdot 3162)^2$.
4. $(\cdot 03961)^3$.
5. $\sqrt{4\cdot 086}$.
6. $\sqrt{\cdot 4876}$.
7. $\sqrt[3]{\cdot 2187}$.
8. $\sqrt[6]{\cdot 008761}$.
9. $(\cdot 5613)^{\frac{2}{3}}$.
10. $(\cdot 0879)^{\frac{3}{5}}$.
11. $(\cdot 126)^2 \times (\cdot 016)^2$.
12. $(\cdot 157)^2 \times (\cdot 029)^4$.
13. $(\cdot 156)^{\frac{2}{3}} \times (\cdot 087)^{\frac{1}{5}}$.
14. $(\cdot 086)^{\frac{3}{5}} \div (\cdot 185)^{\frac{2}{3}}$.
15. $\sqrt{(1\cdot 56)^3} \times \sqrt[3]{(1\cdot 56)^2}$.
16. $\sqrt[5]{(361)^2} \div \sqrt[3]{(\cdot 562)^2}$.
17. $\sqrt[3]{\cdot 07096} \div 865$.
18. $(\cdot 0937)^{\frac{1}{5}} \div \cdot 653$.
19. $0\cdot 651 \times 23\cdot 79 \div 0\cdot 0987$.
20. $(200)^{\frac{3}{4}}$.
21. $(0\cdot 002)^{\frac{3}{5}}$.
22. $(\frac{283}{721})^3$.
23. $(2\cdot 135)^{\frac{2}{3}}$.
24. $\dfrac{7\cdot 689 \times \cdot 0358}{17\cdot 62}$.
25. $\left(\dfrac{1}{\cdot 003}\right)^{\frac{4}{3}}$.
26. $(18\cdot 63)^{\frac{2}{5}}$.

27. $\dfrac{1}{266 \times \cdot 1494}$.

28. $\frac{1}{3}\pi(2\cdot 062)^2 \times 1\cdot 786$ ($\pi = 3\cdot 142$).

29. $\frac{4}{3}\pi(3\cdot 096)^3$.

30. $2\pi\sqrt{\dfrac{15\cdot 69}{32\cdot 04}}$.

EXERCISE 57.

Use logs where necessary to work the following examples:

1. The area of a triangle is given by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s = half the sum of the sides a, b and c.

Find the area when $a = 5\cdot 612''$, $b = 4\cdot 879''$, $c = 3\cdot 105''$.

2. The amount at compound interest of a principal P for n years at r °/ₒ is given by the formula $A = PR^n$, where R is the amount of £1 in 1 year, i.e. $R = 1 + \dfrac{r}{100}$. Using this formula, find the amount of £12 for 25 years at 5 °/ₒ compound interest.

3. If the population of a town increases from 1,896,000 to 1,953,000, find the rate per cent. of increase.

4. What is the simple interest on £125. 10*s*. for 4 years 221 days at $3\frac{1}{4}$ °/ₒ?

5. The volume of a cylinder is given by $V = \cdot 7854\, D^2 L$, where D is the diameter and L the length of stroke. Find D when $V = 156\cdot 2$ and $L = 5\cdot 605$.

6. Find the value to 3 significant figures of

$$\frac{1{,}256{,}000\,(45 - 30\cdot 56)}{1276 \times 6\cdot 333 \times 2\cdot 06}.$$

Caution! Logs can only be used for multiplication, division, roots, and powers, and not for addition and subtraction.

7. The hypotenuse of a right-angled triangle is $5\cdot 962''$, and one side is $3\cdot 106''$. Find the other side, using $c^2 = a^2 + b^2$, where c is the hypotenuse. **Caution!** Use the logs for finding squares and square root only.

8. If $T=2\pi\sqrt{\frac{l}{g}}$ gives the time of swing of a simple pendulum of length l, g being the acceleration of gravity, find T when $\pi=3{\cdot}142$, $l=9{\cdot}87$ and $g=32$.

9. Find the value of $(6{\cdot}7341)^2-\frac{1}{(6{\cdot}7341)^2}$.

10. Find the value of $\sqrt{\frac{17{\cdot}38}{5{\cdot}737}}\times 1{\cdot}738$.

11. The velocity of sound in air at a temperature $t°$ Centigrade in metres per second is given by the formula

$$v=332{\cdot}4\times\sqrt{1+0{\cdot}00366t}.$$

Find the velocity of sound at a temperature of 25° C.

[Note that $\cdot 00366t$ must be evaluated before adding to 1.]

12. We can calculate the time of swing (t) in seconds of a simple pendulum of length l feet, by using the formula

$$t=2\pi\sqrt{\frac{l}{g}}.$$

Given that $g=32{\cdot}2$ and $\pi=3\frac{1}{7}$, find the time of swing of a pendulum 4 feet long.

13. The following formula is often used for finding the pressure of water on a dock gate:

$$D=\frac{P}{0{\cdot}278A},$$

where P is the force on the gate in cwt., A the area exposed to the water in square feet, and D the depth of water in feet. Find the depth of water when the area of the dock gate exposed is 600 sq. ft., and the total force is 3880 cwt.

14. The formula for finding the diameter of a balloon which is necessary in order that it may raise a given weight is

$$D=\sqrt[3]{\frac{W}{0{\cdot}5236\,(A-G)}}.$$

Find D when $W=2560$, $A=0{\cdot}0807$, $G=0{\cdot}0056$.

15. If $H=\frac{I^2Rt}{1058}$, find H when $I=12{\cdot}5$, $R=12{\cdot}6$, $t=300$.

GENERAL REVISION

EXERCISE 58 (a). MENTAL

1. Solve the equation $\frac{2x}{3}=\frac{8}{9}$.

2. I buy eggs at x pence per dozen and sell them at d pence each. What profit in shillings do I make on a gross?

3. Find the value of $\cdot 8x-\cdot 3y$ when $x=5$ and $y=10$.

4. Reduce $\frac{x}{x+1}=\frac{x-1}{2x}$ to a non-fractional form by "multiplying across," i.e. by multiplying both sides by $2x(x+1)$.

5. Change the equation
$$3(x-2)-4(x-3)=x(x-1)-4(x-3)$$
to a simpler form.

6. Simplify $\frac{x}{2}-\frac{y}{3}+\frac{5}{6}$.

7. Find the factors of x^2+3x+2.

8. Solve the equation $(2x+5)(3x-2)=0$.

9. How many dozen are there in x gross?

10. Write down the equation which represents the following: "Divide the number 16 into two parts, so that the square of the first is 40 more than the product of the two parts."

EXERCISE 58 (b).

1. Solve the equations $x+\frac{2}{3}y=\frac{1}{3}$, $x+\frac{3}{4}y+\frac{1}{4}=0$.

2. Find the factors of $6a^2+5a-4$.

3. Solve the equation $\frac{2x-3}{4x+1}=\frac{x-7}{2x-2}$.

4. Simplify $\frac{a^2}{bc}-\frac{b^2}{ac}-\frac{c^2}{ab}$.

5. If $5x+7y=8$, write down the corresponding values of x when $y=-2, 0, 2, 4$.

6. Find the value of $2\pi\sqrt{\frac{l}{g}}$ when $\pi = 3{\cdot}14$, $l = 53{\cdot}8$, $g = 32$. Give the answer correct to the first place of decimals.

7. Subtract $\frac{1}{2}(a + 2b - c)$ from $\frac{1}{3}(a - 2b + c)$

8. Show graphically that the equation $4x^2 + 3x + 12 = 0$ has no real roots.

9. Solve the equation $x = \frac{x+8}{x+3}$.

10. A and B share £40. If A spends half his share, he still has £5 more than B. How much has each at first?

EXERCISE 59 (a). Mental

1. If $x = 2$ satisfies the equation $\frac{ax}{7} = 14$, find the value of a.

2. If I travel uniformly at x miles an hour, how long shall I take to cover 4 miles?

3. If $ax + by = 7$ and $ax - by = 3$, find the value of ax.

4. Reduce to a non-fractional form the equation
$$\frac{3x}{x+1} = \frac{x-1}{2x}.$$

5. Change the equation $\frac{356}{x-2} = \frac{712}{x-3}$ to a simpler form containing no fractions.

6. Solve the equation $\frac{3}{2x} = \frac{2x}{27}$.

7. Find the factors of $4x^2 + 8x$.

8. "Complete the square" in the following equation:
$$x^2 - \tfrac{3}{2}x = \tfrac{7}{4}.$$

9. If a lbs. cost b pence, what will 5 lbs. cost?

10. Write down the equation which will solve the problem:
"The area of a rectangle is 30 square feet and one side is 1 foot longer than the other. Find the lengths of the sides."

EXERCISE 59 (*b*).

1. Solve the equations $x-3y=3x-10y=1$.
2. If $x-1$ is one factor of $x^3-13x+12$, what is the other?
3. Simplify $\frac{3}{5x}+\frac{2}{3x}-\frac{x}{15}$.
4. Solve the equation $(3-a)(1+2a)=7-a^2$.
5. Multiply $3x^2+5x-3$ by $2x-3$.
6. Bring the sum of £x, v crowns, and z sixpences to pence.
7. Solve the equation $\frac{5}{8}(3x-7)-\frac{3}{4}(2x-3)=\frac{1}{2}(x-4)$.
8. Solve graphically the equations given in Question 1.
9. Write down algebraically: Square the sum of a and b, and divide the result by the sum of the squares of a and b.
10. Three customers enter a shop to buy eggs. The first buys 6 more than a third, the second half the remainder, and the third two-thirds of that remainder. If 15 eggs are left, how many were there at first?

EXERCISE 60 (*a*). MENTAL

1. Solve the equation $-\frac{3x}{4}=2$.
2. How far can I go in t minutes at the rate of m miles an hour?
3. If $4x=7y+8$, find the value of y in terms of x.
4. Reduce to non-fractional form the equation
$$\frac{3x+2}{2x-3}=\frac{3x-5}{4x+7}.$$
5. If $\frac{5x-2}{3}-\frac{4x-7}{4}+x-2$ has to be brought to a denominator 12, perform the first line of the operation.
6. Reduce to lowest terms $\frac{3x}{6x^2+9x}$.
7. Find the factors of $x^2+9x+20$.
8. Solve the equation $(x+1)(x+2)(x+3)=0$.
9. Find the Simple Interest on £$100x$ for y years at z %.

10. Write down the equations which will solve the problem: "The perimeter of a rectangle is 26 inches and the area 40 square inches. Find the lengths of the sides."

EXERCISE 60 (*b*).

1. Solve for s: $\dfrac{1-s}{4} - \dfrac{6-2s}{3} = 2s - \dfrac{23}{24}$.

2. Find the factors of $10x^2 - x - 3$.

3. Substitute $1 + x$ for y in the expression $y^2 + 3y + 5$.

4. Simplify $a(3a - 2b) - b(3b - 2a) - (a - b)^2$.

5. What are the three consecutive odd numbers of which the largest is $2x + 1$? If their sum is 63, what is the smallest?

6. Solve the equation $\dfrac{2(x-5)}{4} = \dfrac{3(x-1)}{2} + \dfrac{5(x-3)}{8}$.

7. Obtain a graphical solution of the equations

$$\tfrac{1}{2}x + \tfrac{1}{3}y = 6; \quad \tfrac{1}{3}x - \tfrac{2}{9}y = \tfrac{4}{3}.$$

8. If $y = \frac{1}{2}x^2 - 3x + 2$, find a table of values connecting x and y from $x = -2$ to $x = +2$.

9. In the formula $s = ut + \dfrac{gt^2}{2}$ find s when $u = 42$, $t = \cdot 5$, $g = 32$.

10. If I reduce my motoring speed by 4 miles an hour, I take half an hour longer to do 60 miles. What is my speed?

EXERCISE 61 (*a*). MENTAL

1. Solve the equation $\dfrac{2}{x} = 7$.

2. A fence x yards long is supported by y uprights equally spaced. What is the distance between two adjacent uprights?

3. If $\dfrac{x}{3y} = 7$, find x when $y = 4$.

4. Simplify $\dfrac{x}{x+y} + \dfrac{x}{x-y}$.

5. If $\dfrac{3a-2}{5} - \dfrac{2a-3}{3} = a - 1$, give the first line of the solution when each term is multiplied by 15.

6. Change the equation $2y = \frac{x-7}{2}$ to a non-fractional form in which x and y terms appear on the same side.

7. Find the factors of $x^2 - 5x - 84$.

8. If $(x+2)^2 = \frac{25}{4}$, find two values of x.

9. Increase £x by $2\frac{1}{2}$ °/$_\circ$.

10. Write down the equation which will solve the problem: "Divide a line 3″ long into two parts, so that three times the first is four times the se-ond part."

EXERCISE 61 (*b*).

1. Simplify $(2x - 3y)^2 - 4x(x+y) - 2(3x+y)^2$.

2. What is the square root of $4x^2 - 28xy + 49y^2$?

3. Solve the equation $\frac{2}{x+3} + \frac{3}{x-3} = 1\frac{3}{4}$.

4. If $3s = 4 + 5t$, obtain a table of corresponding values of s and t, from $t = -2$ to $t = +2$.

5. Simplify $\frac{3}{5a} - \frac{2}{5b} + \frac{6}{5c}$.

6. If I travel half a journey of x miles at the rate of y miles an hour, and half at z miles an hour, how long shall I take?

7. Solve the equation $\cdot35x + \cdot5(2x-3) = \cdot7(x-2)$.

8. Find graphically if the equation $2x^2 - 3x + 4 = 0$ has any real roots.

9. An article bought for £x is sold for £5. 5*s*. at a profit of x per cent. Find x.

10. A man's age is five times the sum of the ages of his two children, and in 8 years it will be twice the sum. What is the man's age?

EXERCISE 62 (*a*). MENTAL

1. If $5(x-1) = 8(x-1)$, what must be the value of x?

2. Write down an expression for the perimeter of a rectangle if one side is $\frac{1}{2}a$ and the other $\frac{1}{3}b$.

3. The area of a trapezoid is given by $\frac{1}{2}h(a+b)$, where a and b are the lengths of the parallel sides and h the distance between. What is the area if the parallel sides 7·3″ and 2·7″ are ·5″ apart?

4. Simplify $\frac{1}{x} - \frac{1}{x+1}$.

5. Find the coefficient of x^2 in $(2x+3)(3x+4)-(x-2)(2x+3)$.

6. Solve for x: $2x - 7y = 15$, $3x - 7y = 3$.

7. Divide x^{27} by x^{24}.

8. Solve the equation $2x(5x+7) = 0$.

9. If I was x years old y years ago, how old shall I be in y years time?

10. Write down the equation which will solve the problem. "Divide the number 75 into two parts such that the square of one is 25 more than 12 times the other."

EXERCISE 62 (*b*).

1. Simplify $2a(3a+b) - 3b(2a+3b) - (a+2b)^2$.

2. Find the factors of $21x^2 + 29xy - 10y^2$.

3. Solve the equation $\frac{x}{x+2} = \frac{5}{x+1}$, giving the roots correct to two places of decimals.

4. The sides of a triangle are x, $2x+2$ and $2x+3$ inches in length, and the square on the longest side equals the sum of the squares on the other two sides. Find x.

5. Solve the equation $\cdot 2x + \cdot 3(4-2x) + \cdot 6(5+3x) = 0$.

6. Obtain a graphical solution of the equation

$$5x^2 - 6x = 63.$$

7. Write down algebraically: Add 4 to the square of x, take away the square of y, multiply the whole result by 7.

8. Solve by formula the equation $\frac{1}{x} = \frac{x+2}{3}$.

9. x boys have an average of 5*d.* each. 5 of them have an average of 2*d.* each. What average have the rest?

10. An increase of 2*d.* in the price of a lb. of apples means that I get 2 lbs. less for 4*s.* How much are they per lb.?

EXERCISE 63 (a). MENTAL

1. What is one obvious root of the equation

$$2(x-3)+4x(x-3)=2(x-3)?$$

2. If $X=2l-3m$ and $Y=2l+3m$, what is the value of $\frac{1}{2}(X+Y)$?

3. Find the value of $\sqrt[3]{ax+by}$ when $a=5$, $b=4$, $x=3$, $y=3$.

4. Remove brackets from $-3(s+4t)+2(3s-2t)-5(s+3t)$.

5. Simplify $\dfrac{\dfrac{x}{y}+\dfrac{y}{x}}{\dfrac{x}{y}-\dfrac{y}{x}}$ by multiplying top and bottom by xy.

6. Find y in terms of x, if $-5x+7y=3$.

7. Divide $2y(y^2-3y)$ by y^2.

8. If $12a^2-16ab-3b^2$ is exactly divisible by $2a-3b$, what is the quotient?

9. Find the average of the quantities $2a$, $3b$, $4c$, $4a$, $3b$, $2c$.

10. Write down the equation that will solve the problem:
"The sum of the reciprocals of two consecutive even numbers is $\frac{5}{12}$. Find the numbers."

EXERCISE 63 (b).

1. Simplify $(5x-y)^2-(x-y)^2+(2x+3y)^2$.

2. Solve the equations $\cdot4x+\cdot3y=3\cdot8$, $\cdot6x+\cdot5y=6$.

3. Find the value of $\left(\dfrac{p}{q}-\dfrac{q}{p}\right)\left(\dfrac{p}{2q}+\dfrac{2q}{p}\right)$ when $p=\dfrac{1}{2}$, $q=\dfrac{1}{3}$.

4. For what value of x is $(x+3)(x+4)$ equal to x^2?

5. A square prism has a volume of 250 cubic inches, and the height is twice the length of a side of the base. What is the height?

6. Solve the equation $\frac{4}{5}(2x+3)+\frac{3}{4}(x-2)-\frac{1}{10}(x+3)=0$.

7. Obtain a graphical solution of the equations

$$2x+3y=7,\quad x-2y=0.$$

8. If $s=10t^2$, find s when $t=1\cdot2$ and also when $t=1\cdot02$. Add the results.

9. If $\frac{1}{x^2-7x+12}=\frac{A}{x-4}+\frac{B}{x-3}$, what must be the values of A and B?

10. £x is increased by $10x$ °/ₒ, and the result is equal to £2. 8s. 0d. Find x.

EXERCISE 64 (*a*). MENTAL

1. Simplify $\frac{4(x-7)}{2x-14}$.

2. Solve the equation $\frac{1}{x}+\frac{1}{2x}=\frac{3}{4}$.

3. Find the value of $4x(x+z)(x+3z)$ when $x=\frac{1}{2}$ and $z=0$.

4. Change $\frac{3x+8}{x+2}$ to a simpler form by division.

5. What is the coefficient of x^2 in the product of $4x^3+2x^2+3$ and x^2+x-2?

6. If $3x-7y=-5$, find x in terms of y.

7. What is the square of $3l-5m$?

8. Solve the equation $(5x+7)(3x-8)=0$.

9. If I can row x miles an hour up stream, and y miles an hour down, what is my rate in still water?

10. Write down the equations that will solve the problem:
"A certain fraction is equal to $\frac{1}{2}$, but if I take 2 from the numerator and 1 from the denominator it becomes equal to $\frac{1}{3}$. Find the fraction."

EXERCISE 64 (*b*).

1. Simplify $(x^2-x-1)(x-1)-(x^2+x+1)(x+1)$.

2. Solve the equation $\frac{5}{2x+1}=\frac{x-1}{3x+2}$, giving answers correct to one place of decimals.

3. If $15x^3-ax^2+5x-6$ is exactly divisible by $5x^2+2x+3$, what is the value of a?

4. Add together $\frac{x}{x-2}$, $\frac{x}{x+2}$ and $\frac{x}{(x-2)(x+2)}$.

5. What is the total surface area of a square pyramid if the base is a square of side x inches, and the slant height of each triangle is equal to the length of the base?

6. Solve the equation $\cdot 56x + \cdot 25(4x - 6) = \cdot 36(x - 3)$.

7. Find the sum of $5(2x - y) + 3z$ and $3(y - 2x) - 2z$.

8. If $3x - 2y = 6$, find y in terms of x. Substitute this value in $x + y = 12$, and hence find a pair of values for x and y.

9. Solve graphically $3x^2 - 2x - 5 = 0$.

10. A train does a certain iourney in 5 hours. Another train which travels 10 miles an hour slower takes $1\frac{1}{4}$ hours longer. What is the length of the journey?

EXERCISE 65 (*a*). MENTAL

1. Solve the equation $2x + \frac{1}{2} = 2\frac{1}{2}x + \frac{1}{4}$.

2. Simplify $(\cdot 7a + \cdot 5b) + (\cdot 2a - \cdot 3b) + (\cdot 1a + \cdot 8b)$.

3. The area of a ring is given by $\pi(R + r)(R - r)$, where R is the outer and r the inner radius. Find the area when π is taken to be $\frac{22}{7}$, $R = 4$, $r = 3$.

4. Simplify $\dfrac{1}{x+1} + \dfrac{1}{x-1}$.

5. Simplify $\frac{2}{3}x + \frac{3}{5}y = 2$.

6. What figure is formed when the points 0, 0; 4, 0; 4, 4; and 0, 4 are joined in the order shown?

7. Multiply $-3x + 5y$ by $3x - 5y$.

8. If $7x + 5$ is one factor of $14x^2 - 11x - 15$, what is the other?

9. What is the price of 5 lbs. of apples at x pence a lb.?

10. Write down the equation that will solve the problem:
"If the price of apples is reduced by 1*d*. a lb. I shall get 2 lbs. more for 2*s*. What is the price per lb.?"

EXERCISE 65 (*b*).

1. Multiply $(2x-3)^2$ by $x-2$.

2. Simplify $\dfrac{\dfrac{1}{x}-\dfrac{1}{x^2}}{\dfrac{1}{x}+\dfrac{1}{x^2}}$.

3. Solve the equation $\dfrac{2+a}{3}=\dfrac{2a-3}{2}-\dfrac{2a+3}{5}$.

4. If $\cdot 5x+\cdot 3y=2$, obtain a table of 4 pairs of corresponding values of x and y.

5. Obtain graphically the roots of the equation
$$3x^2-7x-3=0.$$

6. What is the total cost of x lbs. of pears at 1*s.* for y lbs., and y lbs. of apples at 1*s.* for x lbs.?

7. Take $5x-3(2y+z)$ from $3x-5(2y-z)$.

8. Simplify $a(4a+2b+c)-(b+c)(a+b)$.

9. Write down algebraically the stages in the following: Think of a number, double it, add 6 more than the number you thought of, divide by 3, and then take away the number you thought of. What is the result?

10. A builder pays £120 a week in wages. If he employed 5 men more, and paid them 10 shillings a week less, his wages bill would be reduced by £7. 10*s.* 0*d.* How many men did he employ?

EXERCISE 66 (*a*). MENTAL

1. Simplify $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a}-\dfrac{1}{b}}$.

2. Add together $\frac{1}{2}m-\frac{1}{3}n$, $\frac{1}{3}m+\frac{1}{2}n$, $\frac{1}{6}m+\frac{5}{6}n$.

3. Find the value of $\sqrt{x+1}+\sqrt{2x+9}$ when $x=8$.

4. Simplify $3(2c-4d)-2(3e-4f)$.

5. If $\dfrac{1}{x}+\dfrac{2}{y}=7$ and $\dfrac{1}{x}-\dfrac{2}{y}=1$, find the value of x.

6. When the points $-2, 3$; $-2, -3$ are plotted, how many units are they apart on the graph?

7. Find the product of $2a-7b$ and $5a+7b$.

8. Solve the equation $(2x-3)(7x+3)=0$.

9. If I walk at the rate of x miles an hour, how many minutes shall I take to walk 5 miles?

10. Write down the equation that will solve the problem:
"If I change my rate of walking from 5 to 4 miles an hour, I shall take $2\frac{1}{2}$ hours longer to complete my journey. What is the length of the journey?"

EXERCISE 66 (*b*).

1. Solve the equations $\cdot 6x + 2y = -\cdot 54$; $6x - y = \cdot 9$.

2. Obtain a graphical solution of the equation $2x^2 - 3x - 2 = 0$.

3. Simplify $\dfrac{2(x+3y)}{15} + \dfrac{x-2y}{10} - \dfrac{x+y}{5}$.

4. Find the value of $\left(\dfrac{x+a}{x-a} - 1\right)\left(\dfrac{x-a}{x+a} + 1\right)$ when $x = 2$, $a = 1$.

5. Find the factors of $165x^2 + 10x - 15$.

6. Solve the equation $\dfrac{5}{x} - \dfrac{5}{x+1} = \dfrac{1}{2x}$.

7. Solve the equation $\frac{3}{4}(x-3) - \frac{2}{3}(x-4) = \frac{5}{3}(x-5)$.

8. Simplify $(a-3b)^2 + 2a(2a+5b) - (a+3b)^2$.

9. Write down algebraically: Square a, take away twice the square of b, add 3 and then divide the result by the sum of a and b.

10. The area of a plot is reduced by 2 sq. feet if the length is increased by 2 feet and the breadth reduced by 1 foot. Find the length and breadth, if the sum of the length and breadth is 15 feet.

LOGARITHMS

OF

NUMBERS

LOGARITHMS OF NUMBERS.

											Differences.								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS OF NUMBERS.

	0	1	2	3	4	5	6	7	8	9	Differences. 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	3459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	1	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

ANSWERS

EXERCISE 1.

1. 5.	2. 2.	3. 10.	4. 1.	5. 5.	6. 1.
7. -2.	8. -3.	9. -1.	10. 4.	11. 3.	12. 3.
13. 0.	14. 3.	15. -2.	16. -3.	17. 4.	18. -2.
19. 18.	20. 21.	21. 0.	22. -12.	23. 24.	24. 0.
25. 2.	26. $\frac{1}{5}$.	27. -2.	28. 3.	29. $2\frac{1}{2}$.	30. 25.
31. 6.	32. -5.	33. 0.	34. 3.	35. $-\frac{1}{2}$.	36. 1.
37. $\frac{1}{2}$.	38. $-\frac{1}{2}$.	39. $1\frac{1}{2}$.	40. -6.	41. 3.	42. -25.

EXERCISE 2.

1. $17x$.	2. $3a$.	3. $22x$.	4. $12a+6b$.
5. $7a+9x$.	6. $2a+2b+2c$.	7. $1{\cdot}2x+1{\cdot}2y$.	8. $11a$.
9. $12x-y$.	10. $11a+9b$.	11. $15x+5y+9z$.	12. $9x$.
13. 24.	14. $3x+3y$.	15. 50.	16. $1\frac{2}{3}x$.
17. $\frac{1}{3}y$.	18. $1-3x$.	19. $a+b$.	20. $2a$.

EXERCISE 3.

1. 19.	2. 9.	3. 7.	4. $\frac{3}{4}$.	5. 11.	6. 28.
7. 2.	8. 10.	9. 12.	10. 22.	11. $4\frac{2}{3}$.	12. 65.
13. 343.	14. 77.	15. 72.	16. 4.	17. 12.	18. 36.
19. 216.	20. 37.	21. Each side $=13$.		22. Each side $=7$.	

EXERCISE 4.

1. 4.	2. 4.	3. 2.	4. 3.	5. 5.	6. 2.
7. -2.	8. $\frac{1}{2}$.	9. $-\frac{2}{3}$.	10. $\frac{2}{3}$.	11. 1.	12. $1\frac{4}{7}$.
13. 0.	14. 0.	15. $-\frac{4}{5}$.	16. 2.	17. -3.	18. $-\frac{1}{2}$.
19. $2\frac{2}{3}$.	20. $1\frac{5}{6}$.	21. -3.	22. $-\frac{2}{3}$.	23. $1\frac{1}{4}$.	24. 1.

EXERCISE 5.

1. $x+3$.	2. $7-x$.	3. $x+10$ years.
4. $x-10$ years.	5. $3x$ years.	6. $12x$ pence.
7. $20A$ shillings.	8. $x, x+1, x+2$.	9. $x-1, x, x+1$.
10. $2x+3$.	11. $x+10$.	12. $24-x$.
13. $(2l+2b)$ feet.	14. $4n$ inches.	15. $5\frac{1}{2}a$ shillings.
16. £$(6x+7)$.	17. $(20x-y-z)$ shillings.	18. $4x$ pence.
19. $\frac{x}{4}$ miles.	20. $x+8$ miles per hour.	

EXERCISE 6.

1.	8.	2.	6.	3.	13 years.	4.	12.
5.	4 inches.	6.	5, 8.	7.	16.	8.	4 lb.
9.	20, 30.	10.	10.	11.	20 years.	12.	20, 25.
13.	2*s.* 6*d.*	14.	£7, £18.	15.	£15; 30*s.*; 45*d.*	16.	35.
17.	2, 8.	18.	£5, £10, £20.			19.	7 in.; 8 in.

20. 140 boys, 180 girls, 280 infants.

EXERCISE 7.

1.	6.	2.	8.	3.	$\frac{2}{3}$.	4.	$1\frac{1}{3}$.	5.	$1\frac{1}{2}$.
6.	10.	7.	-4.	8.	4·5.	9.	6.	10.	10.
11.	20.	12.	$\frac{5}{8}$.	13.	12.	14.	$-4\frac{1}{2}$.	15.	$2\frac{2}{11}$.
16.	10.	17.	$\frac{6}{11}$.	18.	15.	19.	48.	20.	$11\frac{7}{11}$.

EXERCISE 8.

1.	19.	2.	0.	3.	$4a$.	4.	$2x+2$
5.	$3x+3$.	6.	$a-5b$.	7.	$-4y$.	8.	$2e$.
9.	$-2a+b$.	10.	$-5p-q$.	11.	$5x+3$.	12.	-25.
13.	$6a+5b-c$.	14.	$y-z$.	15.	$2x+8y-4z$.	16.	-2
17.	$1-{\cdot}5x-{\cdot}2y$.	18.	$-2x+3y-4z$.	19.	$4+x$.	20.	0.

EXERCISE 9.

1.	$6+2x$.	2.	$9-6x$.	3.	$25x-10y$.	4.	$-3x-9$.
5.	$-4x+8$.	6.	$-9x+18$.	7.	$5x-12$.	8.	$17x-10$.
9.	$19-8x$.	10.	$-5x$.	11.	$-{\cdot}25x$.	12.	$1-3x+y$.
13.	-1.	14.	$10-4x$.	15.	$11q+32r$.	16.	$-3m+16n$.
17.	$4x+12y+4z$.	18.	$-2t-6r$.	19.	$7a+2b$.	20.	$13x-17$.

EXERCISE 10.

1.	2.	2.	7.	3.	0.	4.	$6\frac{1}{2}$.	5.	$2\frac{2}{5}$.
6.	$-8\frac{1}{2}$.	7.	$-\frac{1}{2}$.	8.	6.	9.	$2\frac{5}{8}$.	10.	$2\frac{2}{3}$.
11.	$1\frac{1}{2}$.	12.	$2\frac{3}{4}$.	13.	1.	14.	3.	15.	-1.
16.	$1\frac{1}{5}$.	17.	$1\frac{3}{5}$.	18.	$1\frac{1}{2}$.	19.	1.	20.	$2\frac{1}{2}$.

EXERCISE 11.

1.	$\frac{10x-17}{12}$.	2.	$-\frac{1}{4}$.	3.	$\frac{11x-1}{12}$.	4.	$\frac{5x+4}{10}$.
5.	$\frac{13x+3}{12}$	6.	$\frac{12-5x}{12}$.	7.	$\frac{2}{3}$.	8.	$\frac{3x-5}{4}$.
9.	$\frac{3-2x}{2}$.	10.	$\frac{9-26x}{20}$.	11.	$\frac{19x+2}{12}$.	12.	$\frac{2x-5}{12}$.

13. $\frac{7-x}{4}$. 14. $\frac{41-7x}{12}$. 15. $\frac{2x+7}{8}$. 16 $\frac{13x-4}{10}$.

17. Each $=\frac{x+1}{12}$. 18. Each $=-\frac{x}{10}$. 19. Each $=\frac{x+2}{6}$.

20. Each $=\frac{35x-30}{12}$.

EXERCISE 12.

1. $-3\frac{1}{2}$. 2. 0. 3. 3. 4. $-\frac{4}{7}$. 5. 3.
6. $2\frac{3}{7}$. 7. 23. 8. $5\frac{5}{14}$. 9. 2. 10. 0.
11. $2\frac{3}{4}$. 12. 11. 13. $2\frac{2}{5}$. 14. $-1\frac{1}{5}$. 15. 12.
16. $-4\frac{2}{7}$. 17. 15. 18. $\frac{5}{7}$. 19. $\frac{1}{24}$. 20. $-2\frac{1}{17}$.
21. 4. 22. $\frac{1}{15}$. 23. $-\frac{1}{20}$. 24. $1\frac{23}{30}$. 25. $-\frac{5}{6}$.

EXERCISE 13.

1. 150. 2. 1200. 3. £6.
4. 14, 15, 16. 5. 20 years, 40 years. 6. £2000.
7. £12, £20, £24. 8. 27 miles, 36 miles. 9. 27 eggs.
10. 4 florins, 10 half-crowns. 11. Father 66 years, son 22 years.
12. 5 feet. 13. 1920, 1280. 14. 20, 60, 80.
15. 42, 54. 16. 30 cows. 17. £60.
18. £20, £20, £20. 19. £200. 20. 540.

EXERCISE 14 (a).

1. $8x-6$ or $-6+8x$. 2. 0. 3. $a+2b$.
4. $\frac{7x}{12}$. 5. 2. 6. $3a-2b$.
7. $12m+4n$. 8. $\frac{10q-14}{30}$. 9. $\frac{a-2}{3}$.
10. $x+3(x+1)=27$.

EXERCISE 14 (b).

1. 24. 2. 5. 3. $\frac{7}{8}m+\frac{5}{8}n$. 4. 10.
5. $(x+y+5)$ years. 6.

x	-2	-1	0	1	2
y	50	43	36	29	22.

7. $v=20$.
8. $3x-19y+18z$. 9. $x=\frac{11}{41}$.
10. 1 mile per hour, 5 miles per hour.

EXERCISE 15 (a).

1. $5a+8b$. 2. $3l-6m-5n+10p$. 3. $x=0$.
4. $\frac{6x-7y}{10}$. 5. $x=2$. 6. 6. 7. $2a+c$.
8. $x=2$. 9. $140x$. 10. $\frac{100}{x}+\frac{100}{2x}=5$.

EXERCISE 15 (*b*).

1. Each $=8a+8b+8c$. 2. $x=3$. 3. $20x-20y-10z$.

4.

x	2	3	4	5	6
y	-15	-13	-11	-9	-7.

5. $r=24$.

6. $4y-8z$. 7. $S=-2\frac{2}{5}$. 8. $\dfrac{5+10x}{12}$.

9. Coefficient $=7$ or -7. 10. 15, 20.

EXERCISE 16 (*a*).

1. $r=3$. 2. $3l-6m-2n+6q$. 3. $x=1\frac{4}{5}$.

4. Coefficient $=9$. 5. $5y-1$. 6. $x=2$.

7. 8. 8. $2x+6y$. 9. $(x+y-z)$ miles.

10. $\frac{x}{2}-2=\frac{29-x}{3}$; or $\frac{x}{3}=\frac{29-x}{2}-2$; or $\frac{x}{3}+2=\frac{29-x}{2}$.

EXERCISE 16 (*b*).

1. $1{\cdot}5x+1{\cdot}5y$. 2. $4x-3$, $4x-1$, $4x+1$, $4x+3$, $4x+5$.

3.

x	0	1	2	3	4	5
y	13	$10\frac{1}{2}$	8	$5\frac{1}{2}$	3	$\frac{1}{2}$

4. $t=14$.

5. $2+a-18b+20c$. 6. $r=\frac{3}{11}$.

7. $\dfrac{19x-14}{8}$. 8. $\dfrac{x-3}{2}$.

9. $-35c-20d+10r$ or $10r-20d-35c$. 10. 25 miles.

EXERCISE 17 (*a*).

1. $a=\frac{5}{12}$. 2. $4x+2y$. 3. $y=7-5x$.

4. $\dfrac{3b}{2a}$. 5. 15. 6. $4a-12b+8c-6$.

7. $2b$. 8. $\dfrac{12q-16}{48}$. 9. £$11x$.

10. $x-\frac{x}{5}-\frac{x}{4}=30$; or $\frac{1}{5}x+\frac{1}{4}x+30=x$ etc.

EXERCISE 17 (*b*).

1. $9s+3t+3r$. 2. $\dfrac{5b}{a}$ shillings. 3. 45.

4. $p=16$. 6. $-2a-13c$. 7. $g=1\frac{6}{7}$.

8. Each $=\dfrac{6x-23}{20}$. 9. $5a+2b-3c$. 10. 10 sixpences; 40 pence.

EXERCISE 18.

1. -5.
2. 7.
3. -3.
4. $-3x$.
5. $a+2b$.
6. $-2x-15y$.
7. $a+3b+c$.
8. $2x+15y-4z$.
9. $7a+306b$.
10. $-10x-30y$.
11. $1-3a+2b+c$.
12. $-3x-5y+2z$.
13. $-3a+2b-5$.
14. $2x+12y-10z$.
15. $5l+68m-33n$.
16. $-2a+2b-4c$.
17. $-7y$.
18. 0.
19. $-3a+2b+4c$.
20. $205x+540y$.

EXERCISE 19.

1. 2.
2. -5.
3. 7.
4. -4.
5. 4.
6. x.
7. $-y$.
8. $6a$.
9. $3b$.
10. $-3c$.
11. $-x-2y$.
12. $-x+5y$.
13. $a+3b$.
14. $2x+9y$.
15. $-2x-9y$.
16. $c+d$.
17. $-2x+4y$.
18. $-10x+27y$.
19. $46a-76b$.
20. $4x+14y$.
21. $42a-45c$.
22. $-10y+72z$.
23. $a-9b+3c$.
24. $3x-5y-8z$.
25. $-x+2y+z$.
26. $3a-8b+2c$.
27. $-3x+5y-3z$.
28. $a+2b-4c$.
29. $-5x+2y-3z$.
30. $-2+2a-4b$.
31. $3a-b-20c$.
32. $5+4y$.
33. $23x-16y-6z$.
34. $3x-11a+2d$.
35. $-b-10$.

EXERCISE 20.

1. $x=5$. $y=1$.
2. $x=6$. $y=1$.
3. $x=6$. $y=2$.
4. $x=2$. $y=1$.
5. $x=1$. $y=2$
6. $x=4$. $y=-2$.
7. $x=4$. $y=2$.
8. $x=3$. $y=-2$.
9. $x=3$. $y=-1$.
10. $x=1$. $y=1$.
11. $x=\frac{1}{3}$. $y=0$.
12. $x=2$. $y=2$.
13. $x=1$. $y=-2$.
14. $x=0$. $y=1$.
15. $x=7$. $y=7$.
16. $x=1$. $y=10$.
17. $x=7$. $y=2$.
18. $x=-25$. $y=5$.
19. $x=2$. $y=3$.
20. $x=701$. $y=201$.
21. $x=-2$. $y=6$.
22. $x=-10$. $y=2$.
23. $x=23$. $y=17$.
24. $x=1$. $y=-2$.
25. $x=3$. $y=-2$.
26. $x=7$. $y=-4$.
27. $x=5$. $y=-7$.
28. $x=24$. $y=24$.
29. $x=4$. $y=1$.
30. $x=2$. $y=3$.

EXERCISE 21.

1. 10 and 7.
2. £6. 10*s*.
3. $\frac{3}{8}$.
4. 72 and 60.
5. 50 sq. inches.
6. 16 half-crowns; 11 shillings.
7. 3600.
8. 3 miles per hour.
9. 80 and 70.
10. Half-crowns and crowns.
11. $\frac{8}{16}$.
12. 120 and 144.
13. 200.
14. 200.
15. *A* 35 years; *B* 25 years.
16. Coffee 1*s*. 2*d*. per lb.; tea 1*s*. 8*d*. per lb.
17. 24.
18. *A* £150; *B* £180; *C* £270.
19. 2 miles.
20. 120, 240, 150, 300.

EXERCISE 22.

1. $x=\frac{3y-7}{2}$.
2. $b=\frac{-2a-4}{3}$..
3. $z=\frac{5-3x}{7}$.
4. $x=\frac{2y+8}{7}$.
5. $a=\frac{2b-5}{7}$.
6. $x=\frac{5y+11}{6}$.
7. $z=\frac{-7}{5}x$.
8. $a=\frac{-4b-2c}{5}$.
9. $y=\frac{x}{7}$.
10. $x=4y$.
11. $x=\frac{y}{6}$.
12. $b=\frac{8a}{3}$.
13. $x=-2y$.
14. $y=\frac{7x+2}{5}$.
15. $x=\frac{14y+33}{3}$.
16. $y=\frac{7-x}{3}$.
17. $x=\frac{5-2a}{2}$.
18. $y=\frac{x}{4}$.
19. $x=6y-3$.
20. $y=\frac{8}{5}$.

EXERCISE 23.

	$x=$	-3	-2	-1	0	1	2	3.
1. $y=\frac{20-3x}{4}$,	$y=$	$7\frac{1}{4}$	$6\frac{1}{2}$	$5\frac{3}{4}$	5	$4\frac{1}{4}$	$3\frac{1}{2}$	$2\frac{3}{4}$.
2. $y=\frac{7x-15}{3}$,	$y=$	-12	$-9\frac{2}{3}$	$-7\frac{1}{3}$	-5	$-2\frac{2}{3}$	$-\frac{1}{3}$	2.
3. $y=\frac{15-3x}{2}$.	$y=$	12	$10\frac{1}{2}$	9	$7\frac{1}{2}$	6	$4\frac{1}{2}$	3.
4. $y=\frac{43-5x}{9}$,	$y=$	$6\frac{4}{9}$	$5\frac{8}{9}$	$5\frac{1}{3}$	$4\frac{7}{9}$	$4\frac{2}{9}$	$3\frac{2}{3}$	$3\frac{1}{9}$.
5. $y=\frac{7-x}{8}$,	$y=$	$1\frac{1}{4}$	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$.
6. $y=\frac{3x-12}{4}$,	$y=$	$-5\frac{1}{4}$	$-4\frac{1}{2}$	$-3\frac{3}{4}$	-3	$-2\frac{1}{4}$	$-1\frac{1}{2}$	$-\frac{3}{4}$.
7. $y=\frac{5x-11}{2}$,	$y=$	-13	$-10\frac{1}{2}$	-8	$-5\frac{1}{2}$	-3	$-\frac{1}{2}$	2.
8. $y=\frac{5x-7}{2}$,	$y=$	-11	$-8\frac{1}{2}$	-6	$-3\frac{1}{2}$	-1	$1\frac{1}{2}$	4.
9. $y=-(2x+1)$,	$y=$	5	3	1	-1	-3	-5	-7.
10. $y=\frac{40-7x}{5}$,	$y=$	$12\frac{1}{5}$	$10\frac{4}{5}$	$9\frac{2}{5}$	8	$6\frac{3}{5}$	$5\frac{1}{5}$	$3\frac{4}{5}$.

EXERCISE 24.

1.	$x=-4$	-1	2	5	$8.$			
	$y=12$	7	2	-3	$-8.$			
2.	$x=-10$	-5	0	5	10	$15.$		
	$y=11$	9	7	5	3	$1.$		
3.	$x=-3$	-2	-1	0	1	2	3	$4.$
	$y=-25$	-19	-13	-7	-1	5	11	$17.$
4.	$x=-4$	-1	2	5	$8.$			
	$y=-19$	-9	1	11	$21.$			
5.	$x=-3$	0	3	6	$9.$			
	$y=-1$	3	7	11	$15.$			
6.	$x=0$	1	$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{2}.$
	$y=-1\frac{3}{4}$	$-1\frac{1}{4}$	-1	$-\frac{3}{4}$	$-\frac{1}{4}$	0	1	$2.$
7.	$x=-4$	-2	0	2	4	$6.$		
	$y=-1$	4	9	14	19	$24.$		
8.	$x=-7$	-3	1	5	$9.$			
	$y=-13$	-6	1	8	$15.$			
9.	$x=-2$	-1	0	1	2	$3.$		
	$y=-5$	-3	-1	1	3	$5.$		
10.	$x=-5$	-1	3	7	11	$15.$		
	$y=-8$	-5	-2	1	4	$7.$		
11.	$x=-6$	-3	0	3	6	$9.$		
	$y=12$	10	8	6	4	$2.$		
12.	$x=-12$	-5	2	9	$16.$			
	$y=-15$	-5	5	15	$25.$			
13.	$x=-6$	-1	4	9	$14.$			
	$y=7$	5	3	1	$-1.$			
14.	$x=20$	17	14	11	8	$5.$		
	$y=0$	1	2	3	4	$5.$		
15.	$x=-10$	-5	0	5	$10.$			
	$y=-7$	-3	1	5	$9.$			
16.	$x=-5$	-2	1	4	7	$10.$		
	$y=-9$	-4	1	6	11	$16.$		
17.	$x=-5$	-1	3	7	$11.$			
	$y=-10$	-5	0	5	$10.$			
18.	$x=-3$	-1	1	3	$5.$			
	$y=8$	5	2	-1	$-4.$			
19.	$x=-4$	-2	0	2	4	$6.$		
	$y=14$	9	4	-1	-6	$-11.$		
20.	$x=-13$	-4	5	14	$23.$			
	$y=-3$	-1	1	3	$5.$			

EXERCISE 25.

7.	$x=$	0	3	5 etc.
	$y=$	9	0	-6.
8.	$x=$	-4	3	10 etc.
	$y=$	-4	1	6.
9.	$x=$	-5	0	5 etc.
	$y=$	4	0	-4.
10.	$x=$	-5	0	5 etc.
	$y=$	-1	1	3.
11.	$x=$	-1	3	7 etc.
	$y=$	1	-2	-5.
12.	$x=$	-3	0	3 etc.
	$y=$	14	7	0.
13.	$x=$	-3	0	3 etc.
	$y=$	-10	1	12.
14.	$x=$	-2	1	4 etc.
	$y=$	12	10	8.
15.	$x=$	-5	3	11 etc.
	$y=$	10	12	14.
16.	$x=$	-2	7	16 etc.
	$y=$	3	-4	-11.

EXERCISE 26.

1. $x=2$. $y=3$.
2. $x=3\frac{1}{2}$. $y=3$.
3. $x=5$. $y=2$.
4. $x=3$. $y=5$.
5. $x=-1$. $y=-2$.
6. $x=3$. $y=-4$.
7. $x=-12$. $y=-6$.
8. $x=-3$. $y=-6$.
9. $x=-3$. $y=0$.
10. $x=20$. $y=76$.
11. $x=3\frac{2}{5}$. $y=1$.
12. $x=0$. $y=-7$.

EXERCISE 27 (*a*).

1. $x=\dfrac{7-5y}{2}$.
2. $y=1$.
3. $a+2l+2k$.
4. $x=8$.
5. $6a-4b-10c-15d$.
6. $\dfrac{x}{2y+3m}$.
7. $x=1$.
8. $y=\dfrac{3x}{7}$.
9. $\dfrac{7x}{12}$ pence.
10. $x+y=35$; $\dfrac{x}{y}=\dfrac{3}{2}$.

EXERCISE 27 (*b*).

1. $x=3$; $y=2$.
2. $s+2t-3r$.
3.

$x=$	-4	3	10	17.
$y=$	6	2	-2	-6.

4. Each $=\dfrac{x+14}{12}$.
5. $x=4$.
6. $x=\dfrac{2y-1}{2}$.
7. $23-27x$.
8. $\dfrac{31-59x}{12}$.
9. $\dfrac{5+x}{24}$ pence each.
10. 35 and 40 years.

EXERCISE 28 (*a*).

1. $x+2y=3$.
2. $x=7$, $y=3$.
3. $c+4m+n$.
4. $\frac{5}{9}$.
5. $\dfrac{13}{6x}$.
6. $\dfrac{a-3b}{4a+6c}$.
7. $x=\dfrac{5}{7}$.
8. $2{\cdot}1a$.
9. $5x$ miles.
10. $x+y=10$; $x=1\frac{1}{2}y$.

EXERCISE 28 (b).

1. $c+\frac{1}{6}d$.
2. $x=7$, $y=7$,
3. $\frac{9l-10m}{24}$.
4. $x=\frac{1}{24}$.
5. 35.
6. $18x-6y-6a$.
7. $\frac{x-2y}{x-5y}$.
8. $x= 1 \quad 3 \quad 5 \quad 7$.
   $y= -6 \quad -1 \quad 4 \quad 9$.
9. $\frac{5x}{y}$ miles.
10. 18 and 10.

EXERCISE 29 (a).

1. $y=2$.
2. $6x-16y=34$.
3. $-3+7q-5r$.
4. 1.
5. $\frac{3}{2a}$.
6. $\frac{4a+d}{2x+y}$.
7. $x=5$.
8. $\frac{5}{6}a+\frac{5}{6}b$.
9. $\frac{x}{x-5}$.
10. $\frac{x}{2}+\frac{x}{3}=2\frac{1}{2}$.

EXERCISE 29 (b).

1. $11a-3b-4c$.
2. $\frac{9x-5y}{12}$.
3. $a=5$, $b=2$.
4. $x=1 \quad 4 \quad 7 \quad 10$
   $y=8 \quad 1 \quad -6 \quad -13$.
5. $x=-5$.
6. $7m-4l-2n$.
7. Each side$=9$.
8. $l=\frac{1-4a}{7}$.
9. $3x+9$.
10. A £48; B £16; C £6.

EXERCISE 30 (a).

1. $4x=3y-9$, or $3y-4x=9$.
2. $y=1$.
3. $-2q+3r-5t$.
4. $3\frac{1}{2}$.
5. $10x-5-15y$.
6. 2.
7. $\frac{4m-6s}{30}$.
8. $a+3b$.
9. $2x-4, 2x-2, 2x$.
10. $3x+6=2(x+6)$.

EXERCISE 30 (b).

1. $14a-12b-7c$.
2. $s=-\frac{17}{25}$.
3. $\frac{21-15x}{14}$.
4. $16x-y$.
5. $r=5$.
6. Each side$=-2$.
7. $\frac{44}{15x}$.
8. $x= -6 \quad -1 \quad 4 \quad 9$
   $y= -5 \quad -3 \quad -1 \quad 1$.
9. $\frac{6a}{b}$ miles.
10. £60.

EXERCISE 31.

1. $12x$.	2. $10a$.	3. $6y$.	4. $2x^2$.
5. $12x^2$.	6. $-3y^2$.	7. $-8y^2$.	8. $-6x^3$.
9. $6x^2$.	10. $5x^2$.	11. $-15x^2$.	12. $21x^2$.
13. x^3.	14. $12x^3$.	15. $8x^2$.	16. $-3y^3$.
17. $-4x^3$.	18. $6xy$.	19. $6xy$.	20. $-12ab$.
21. a^2.	22. $4x^2$.	23. $9y^2$.	24. $25x^2$.
25. abx^2y.	26. c^3x^3.	27. $6a^2b^3x$.	28. $-25x^4y^3$.
29. $-12abx^4$.	30. $75x^5y$.		

EXERCISE 32.

1. $8x+12y$.	2. $15x-10y$.	3. $15x-21y$.
4. $12a+18b$.	5. $-10x+35y$.	6. $-20y+12x$.
7. $12x+20y$.	8. $16a+72b$.	9. $-15x-24y$.
10. $-28b-63c$.	11. $4a^2+8a-28$.	12. $-5x^2-15x-45$.
13. $-6x^2+15x+24$.	14. $-18x^2-54x+42$.	15. $-3x^3+7x^2$.
16. $-16x^3-36x^2$.	17. $-15xy+9y$.	18. $-4ac-6bc$.
19. $-14x^2-6xy$.	20. $-25xy+25y^2$.	21. $-4a^2x^2-4abxy$.
22. $-9bx^3-6bx^2y$.	23. $-6x^3y-8x^2y+4xy$.	24. $-3a^3x-6a^2x^2-3ax^3$.

EXERCISE 33.

1. $2x^2+9x+9$.	2. $6x^2+11x+4$.	3. $3x^2-7x-6$.
4. $6x^2+x-2$.	5. $12x^2-33x+18$.	6. $2x^2-13x+20$.
7. $12-17x+6x^2$.	8. $-24x^2+31x-10$.	9. $2a^2+5ab-3b^2$.
10. $9a^2-4b^2$.	11. $-25x^2+70xy-49y^2$.	12. $-12x^2+14xy-4y^2$.
13. $-21x^2+29x+72$.	14. $21x^2+26x+8$.	15. $10x^2+51x+27$.
16. $-28x^2-37x+21$.	17. $25x^2+10xy+y^2$.	18. $16a^2+8ab+b^2$.
19. $49x^2-42x+9$.	20. $64c^2+16cd+d^2$.	21. $1-6k+9k^2$.
22. $9x^2-42xy+49y^2$.	23. $9l^2+42lm+49m^2$.	24. $4-28x+49x^2$.
25. $64r^2-80r+25$.	26. $49d^2-126d+81$.	27. $16+40x+25x^2$.
28. $9+42x+49x^2$.	29. $9q^2-12qr+4r^2$.	30. $16s^2-8sm+m^2$.

EXERCISE 34.

1. $4a^2-4ab-18b^2$.	2. $7x^2+7xy+2y^2$.
3. $18t^2-24st$.	4. $5x^2-8xy+13y^2$.
5. $-4x^2-17xy-4y^2$.	6. $4x^2+10x-12$.
7. $x^2+15xy-24y^2$.	8. $9x^2-13x+1$.
9. $29x^2+32x+29$.	10. $16a^2+7ab+2b^2$.
11. $6x^3-11x^2-23x+20$.	12. $4x^4+17x^3-30x^2-29x+14$.

13. $4x^4-4x^3+2x^2-3x+1$.
14. $2a^3-5a^2b-4ab^2+7b^3$.
15. $4a^3+8a^2b+ab^2-3b^3$.
16. $x^3+6x^2+12x+8$.
17. $27x^3+27x^2y+9xy^2+y^3$.
18. $64a^3-144a^2+108a-27$.
19. x^3+5x^2+8x+4.
20. $18x^3-75x^2+104x-48$.

EXERCISE 35.

1. $3x$.
2. $2x$.
3. a.
4. x^2.
5. $3a^2$.
6. $3c$.
7. $-9b$.
8. $-4b$.
9. $-5a$.
10. $6x^2$.
11. $19y^2$.
12. $4c^2$.
13. $4a$.
14. $6c$.
15. $3x$.
16. $-6y^2$.
17. $-2a^2$.
18. $-4x^2$.
19. $-x^2$.
20. $6y$.
21. $-3x^3$.
22. $3x^3$.
23. $-3y$.
24. $-3x$.
25. $3b$.
26. $-2x$.
27. $-2y$.
28. $-2ab$.
29. $-xy$.
30. $4z$.

EXERCISE 36.

1. $4x^2+3x+2$.
2. $a-b$.
3. $-5x+7$.
4. $-5x^2+3xy+1$.
5. $-4x^2+2x-3$.
6. $-8x^2+7-3x$.
7. $-3x^2+4x+2$.
8. $-3a+4b$.
9. $5x^2+3x-2$.
10. $t-2cx-4x^2$.
11. $-5x^2+4xy+y^2$.
12. $-2x^2-3x^3-7$.
13. $x-y$.
14. $y-2$.
15. $x-2y$.
16. $-2x+3y$.
17. $-2a+3x$.
18. $-9+6x+8x^2$.
19. $3-s$.
20. $-3+m$.

EXERCISE 37.

1. $(x+3)(x+4)$.
2. $(x+5)(x+6)$.
3. $(x+1)(x+7)$.
4. $(x+1)(x+8)$.
5. $(x+8)(x+5)$.
6. $(x+10)(x+2)$.
7. $(c+8)(c+12)$.
8. $(a+3)(a+9)$.
9. $(m-8)(m-5)$.
10. $(t-7)(t-11)$.
11. $(r-10)(r-5)$.
12. $(q-8)(q-4)$.
13. $(x-11)(x-6)$.
14. $(c-16)(c-10)$.
15. $(x+8)(x-2)$.
16. $(x-7)(x+5)$.
17. $(b-10)(b+8)$.
18. $(x+12)(x-7)$.
19. $(x-15)(x+3)$.
20. $(x+16)(x-2)$.

EXERCISE 38.

1. 4, 7.
2. 3, 8.
3. -6, -4.
4. -8, -9.
5. 2, -6.
6. 8, -5.
7. -4, -10.
8. -9, 7.
9. -8, -5.
10. -11, 8.
11. 55, -42.
12. -56, 87.
13. -29, -64.
14. -77, 33.
15. -108, 106.
16. -73, -63.
17. -2, -2.
18. -9, -9.
19. 11, 11.
20. 55, 55.

EXERCISE 39.

1. 1, 3.	2. 4, 2.	3. 7, 2.	4. 5, 3.
5. $-2, -4$.	6. $-5, -5$.	7. $-1, -2$.	8. $-3, -2$.
9. $9, -1$.	10. $10, -3$.	11. $11, -9$.	12. $7, -4$.
13. $-8, 3$.	14. $-9, 4$.	15. $-4, -3$.	16. $-8, -1$.
17. 8, 5.	18. 10, 16.	19. $-10, 8$.	20. $-12, 7$.
21. $-15, 3$.	22. $-16, 2$.	23. $-8, 2$.	24. $-12, 7$.
25. $-10, 8$.	26. $-7, 5$.	27. $-15, 3$.	28. 8, 5.
29. 5, 10.	30. 3, 9.		

EXERCISE 40.

1. $(2x+3)(3x+2)$.	2. $(3x+7)(7x+8)$.	3. $(5x+1)(3x+8)$.
4. $(7x+2)(2x+7)$.	5. $(4x-5)(3x-4)$.	6. $(2x-5)(2x-11)$.
7. $(5x-7)(7x-11)$.	8. $(5x-7)(3x-5)$.	9. $(2x+5)(x-7)$.
10. $(3x+11)(x-5)$.	11. $(5x-1)(5x+11)$.	12. $(15x+2)(2x-7)$.
13. $(25x-1)(x-1)$.	14. $(23x-3)(3x-1)$.	15. $(12x-5)(15x-7)$.
16. $(11x-13)(13x-11)$.	17. $(12x-5)(15x+8)$.	18. $(5x+7)(25x-3)$.
19. $(8-7x)(3-5x)$.	20. $(5-6x)(7-2x)$.	21. $(3-25x)(7+3x)$.
22. $(25-2x)(5+4x)$.	23. $(5-7x)(5-7x)$.	24. $(7-11x)(7-11x)$.
25. $(4-13x)(4-13x)$.	26. $(9-14x)(9-14x)$.	27. $(3x-11)(3x-11)$.
28. $(4x-5)(4x-5)$.	29. $(12x-7)(12x-7)$.	30. $(15x+13)(15x+13)$.

EXERCISE 41.

1. $\frac{4}{5}, \frac{3}{4}$.	2. $\frac{2}{7}, \frac{3}{8}$.	3. $\frac{11}{4}, 5$.	4. $8, \frac{3}{7}$.
5. $\frac{5}{9}, -\frac{5}{11}$.	6. $\frac{11}{3}, -\frac{11}{7}$.	7. $-\frac{7}{45}, \frac{5}{3}$.	8. $-\frac{17}{5}, \frac{7}{3}$.
9. $\frac{3}{8}, -\frac{19}{3}$.	10. $\frac{37}{45}, \frac{83}{35}$.	11. $\frac{7}{215}, -\frac{17}{235}$.	12. $-\frac{73}{305}, \frac{85}{403}$.
13. $-\frac{11}{75}, -\frac{2}{53}$.	14. $\frac{17}{5}, \frac{17}{5}$.	15. $\frac{15}{7}, \frac{15}{7}$.	16. $-\frac{9}{2}, -\frac{9}{2}$.
17. $\frac{2}{3}, \frac{3}{2}, \frac{4}{3}$.	18. $\frac{17}{3}, -\frac{15}{4}, -\frac{11}{2}$.	19. 0, 7, 8.	20. $0, 15, -\frac{73}{3}$.

EXERCISE 42.

1. $\frac{3}{2}, 4$.	2. $\frac{5}{4}, 3$.	3. $\frac{7}{3}, 2$.	4. $\frac{9}{2}, 1$.	5. $\frac{3}{4}, \frac{1}{2}$.
6. $\frac{2}{7}, \frac{1}{3}$.	7. $\frac{3}{4}, \frac{5}{2}$.	8. $-\frac{4}{3}, 3$.	9. $-\frac{2}{5}, -\frac{3}{2}$.	10. $\frac{3}{7}, -\frac{7}{2}$.
11. $\frac{1}{5}, \frac{2}{3}$.	12. $-\frac{5}{7}, -5$.	13. $-2, -\frac{7}{3}$.	14. $\frac{3}{2}, -2$.	15. $-\frac{5}{2}, \frac{3}{2}$.
16. $\frac{11}{5}, \frac{5}{11}$.	17. $-\frac{4}{9}, \frac{8}{3}$.	18. $-\frac{7}{5}, -\frac{7}{3}$.	19. $\frac{5}{7}, -\frac{11}{8}$.	20. $\frac{3}{7}, \frac{5}{9}$.
21. $\frac{7}{11}, \frac{7}{11}$.	22. $\frac{5}{2}, \frac{11}{2}$.	23. $\frac{7}{5}, \frac{5}{3}$.	24. $-\frac{11}{3}, 5$.	25. $-\frac{11}{5}, \frac{1}{5}$.
26. $1, \frac{1}{25}$.	27. $\frac{5}{12}, \frac{7}{15}$.	28. $\frac{3}{5}, \frac{8}{7}$.	29. $-\frac{7}{3}, \frac{3}{25}$.	30. $\frac{4}{13}, \frac{4}{13}$.
31. $4, \frac{3}{2}$.	32. $3, \frac{1}{3}$.	33. $1, -\frac{4}{3}$.	34. 0, 4.	35. $7, -\frac{21}{8}$.
36. $-3, \frac{1}{2}$.	37. $7, -12$.	38. $4, -5$.	39. $12, -5$.	40. $8, -9$.
41. $1, -\frac{4}{3}$.	42. $2, \frac{1}{7}$.			

EXERCISE 43.

1. 3, 1.
2. $-8, -4$.
3. 13, -7.
4. -16, 8.
5. 15, -8.
6. -11, 8.
7. $\frac{3}{4}, \frac{1}{2}$.
8. $-\frac{4}{3}$, 3.
9. $\frac{7}{3}, \frac{5}{3}$.
10. 1, $\frac{1}{25}$.
11. $-\frac{3}{5}, \frac{5}{3}$.
12. $-\frac{3}{8}, \frac{2}{3}$.
13. $-\frac{1}{3}$, 3.
14. $\frac{5}{3}, -\frac{2}{3}$.
15. $-\frac{3}{4}, \frac{4}{3}$.
16. $\frac{3}{2}$, 5.
17. 2·62, ·38.
18. 5·65, ·35.
19. 4·41, 1·59.
20. 3·73, ·27.

EXERCISE 44.

1. $-\frac{1}{3}$, 2.
2. 1·28, $-$·78.
3. $-\frac{4}{7}$, 1.
4. $\frac{1}{5}$, -3.
5. 13, -4.
6. 7, -2.
7. 4, 1.
8. $\frac{1}{4}, \frac{1}{3}$.
9. 33·45, 1·55.
10. ·52, $-$11·52.
11. $\frac{1}{8}$, -6.
12. $-\frac{3}{8}, \frac{2}{3}$.
13. 4·37, $-$1·37.
14. 1·37, $-$·37.
15. 2·62, ·38.
16. $-\frac{5}{3}$, 4.
17. 4·41, 1·59.
18. 3·73, ·27.
19. $-$1·62, 1.
20. $\frac{1}{2}$, 1.

EXERCISE 45.

1. 2, 5.
2. 3, 5.
3. 12, 6.
4. 15, 16.
5. 5, 2.
6. 13.
7. $\frac{3}{4}, \frac{4}{3}$.
8. 20 and 12 miles per hour.
9. 24 miles per hour.
10. 10, 11, 12.
11. 42 miles per hour.
12. 40 miles per hour.
13. 10 in.; 5 in.
14. $7\frac{1}{2}$ in.; $2\frac{1}{2}$ in.
15. 15*d.*
16. 12 ft.; 15 ft.
17. 24 ft.; 12 ft.
18. $1\frac{1}{2}$ yards.
19. 12 miles per hour.
20. 3*d.* per dozen.

EXERCISE 46 (*a*).

1. $6a^2+5ao-6b^2$.
2. $-4x^2+xy-2y^2$.
3. $(l-3m)(l-2m)$.
4. $x=\frac{5}{3}$ or $-\frac{3}{4}$.
5. Left$=8-10+2=0$.
6. $x=\frac{3}{4}y$.
7. $s=1$.
8. -77.
9. $\frac{n}{m}$ hours.
10. $\frac{x}{30}=\frac{x}{25}-2$; or $\frac{x}{25}-\frac{x}{30}=2$.

EXERCISE 46 (*b*).

1. $-2x^2+6xy$.
2. $2x^3-7x^2+5x-6$.
3. 5, $\frac{4}{9}$.
4. $x=$ $15\frac{1}{2}$ 13 $10\frac{1}{2}$ 8 $5\frac{1}{2}$.
$y=-$ 2 -1 0 1 2.
5. $x=\frac{1}{2}$.
6. $x-1$.
7. $\frac{52}{15x}$.
8. $2+8\frac{1}{2}x-x^2$.
9. $x=4, y=3$.
10. 1.30 p.m.

EXERCISE 47 (a).

1. $-6x^3+9x^2-21x$. 2. $(x-12)(x+7)$. 3. $2x+7$.

4. $y=\frac{4}{3b}$. 5. $x=7$. 6. $8s^2-2st-15t^2$.

7. $\frac{2x+2}{x(x+2)}$. 8. $y=\frac{x}{4}$. 9. $\frac{x}{a}$ times.

10. $1\frac{1}{10}x=1\frac{1}{2}x-50$.

EXERCISE 47 (b).

1. $\frac{4}{21}$. 2. $(5x+4a)(3x-2a)$. 3. $x=5$.

4. $2x^2-21x+5$. 5. $-\frac{2}{3}$, 4. 6. $6x^4-11x^3-7x^2+21x-9$.

7. x^2+3x+4. 8. $\frac{3x-7}{12}$. 9. $x=\frac{2y+4}{3}$.

10. $2\frac{1}{2}$ feet.

EXERCISE 48 (a).

1. $2a^2-2ab-4b^2$. 2. $x=6$. 3. $a=\frac{1}{2}$. 4. $-6a+9b$.

5. $3y+3$. 6. $\frac{3}{2}$. 7. $y=3$. 8. $\frac{2y+5x}{xy}$.

9. $2x-7$. 10. $\frac{24}{x}-\frac{24}{x+2}=1$.

EXERCISE 48 (b).

1. $\frac{13}{6(x-1)}$. 2. $\begin{array}{lrrrrrr} a= & -14 & -7 & 0 & 7 & 14 & 21. \\ b= & -6 & -3 & 0 & 3 & 6 & 9. \end{array}$ 3. $x=\frac{11}{21}$.

4. $y=\frac{1-20x}{6}$. 5. 2, 4. 6. 4, $-\frac{1}{5}$.

7. x^3-6x^2+5x. 8. $a=5$. 9. $\frac{6a+17b}{20}$. 10. 40 miles.

EXERCISE 49 (a).

1. $3x^2-6xy-6xz+4x$. 2. 5. 3. $\frac{3x+1}{x(x+1)}$.

4. $(r-12)(r-3)$. 5. $a=3\frac{1}{2}$. 6. $\frac{x-7y}{2x-3y}$.

7. -6. 8. $\begin{array}{lrrr} x= & -2 & 0 & 2 \\ y= & 4 & 0 & 4. \end{array}$ 9. xy miles.

10. $\frac{x+5}{y+2}=\frac{x}{y}+\frac{3}{10}$; $x+y=17$.

EXERCISE 49 (*b*).

1. $5a^2-16a+13$. 2. $x=-6$. 3. $\dfrac{5p+5q}{(2p-q)(3p+q)}$.
4. $(4x-3)(2x-3)$. 5. $\frac{5}{11}, -\frac{6}{11}$. 6. $12x^2-17x+6=0$.
7. $4x^4-25x^2+20x-4$. 8. $y=-\frac{1}{5}x$. 9. 166.
10. 3 ten shilling notes.

EXERCISE 50.

1. 22·8° C. 2. 5·4 k. 3. 78. 4. 78·5 sq. in.
5. £2250. 6. 4·7. 7. 5·3. 8. 36·5.
11. 51 sq. in. 12. 4·45. 13. 10·4. 14. 30.
15. 8·6. 16. 6·4 cub. in.; 37·6 in. 17. 53.
18. $2\frac{1}{4}$ hours; $12\frac{3}{8}$ miles. 19. 1·4 hours; 13·6 miles.
20. Half-way at 10 a.m. 21. $4\frac{1}{2}$ miles from *A*.
22. 3.27 p.m. 23. 10.24 a.m. 24. 14th.

EXERCISE 51.

11. −2, 3·5. 12. 1, 6. 13. 5·33, −2·5.
14. 2·41, −·41. 15. $\frac{1}{2}$, 2. 16. 5·20, −·20.
17. 1·19, −2·52. 18. 1·92, −·52. 19. ·73, −·27.
20. No real roots.

EXERCISE 52.

1. 3·5272. 2. 3·6241. 3. 3·7024. 4. 1·4097.
5. 2·5055. 6. ·3617. 7. 2·1335. 8. 2·6304.
9. 2·7243. 10. 1·6335. 11. 1·8751. 12. ·9685.
13. ·7782. 14. $\bar{1}$·8451. 15. $\bar{1}$·9542. 16. $\bar{1}$·4950.
17. $\bar{1}$·6304. 18. $\bar{1}$·7211. 19. $\bar{1}$·4448. 20. $\bar{1}$·5139.
21. 0·0000. 22. $\bar{2}$·5752. 23. $\bar{3}$·1818. 24. $\bar{5}$·9345.
25. $\bar{5}$·6990. 26. $\bar{2}$·9325. 27. $\bar{3}$·5704. 28. $\bar{2}$·9355.
29. $\bar{5}$·9312. 30. $\bar{2}$·8451.

EXERCISE 53.

1. 18·03. 2. 121·3. 3. 10·38. 4. ·02298.
5. ·6029. 6. ·001003. 7. ·2818. 8. ·03787.
9. 829·2. 10. ·001049. 11. ·3641. 12. ·005248.
13. 2·969. 14. 102·1. 15. 1·444. 16. ·2040.
17. 7·872. 18. ·02296. 19. ·001004. 20. 1·003.

EXERCISE 54.

1. 87·9. 2. 573. 3. 455. 4. 23·4.
5. 2·30. 6. 1230. 7. 1·02. 8. 1030.
9. ·0270. 10. ·230. 11. ·00741. 12. ·00513.
13. ·00929. 14. ·0154. 15. ·000000723. 16. ·000000203.
17. ·000243. 18. ·00000361. 19. 34·6. 20. ·020.

EXERCISE 55.

1. 1·59.	2. 1·31.	3. 104.	4. 16·1.
5. 7·08.	6. 136.	7. 1·67.	8. 1·76.
9. 1·30.	10. ·245.	11. 425.	12. 1580.
13. 17·6.	14. 2510.	15. 39200.	16. 6·40.
17. 65·6.	18. 190.	19. 8250.	20. 17·0.

EXERCISE 56.

1. 3·85.	2. 1·05.	3. ·100.	4. ·0000622.
5. 2·02.	6. ·698.	7. ·603.	8. ·454.
9. ·681.	10. ·233.	11. ·00000406.	12. ·0000000174.
13. ·128.	14. ·707.	15. 2·62.	16. 15·5.
17. ·000479.	18. ·954.	19. 157.	20. 53·2.
21. ·0240.	22. ·0605.	23. 1·66.	24. ·0156.
25. 2310.	26. 5·78.	27. ·0252.	28. 7·95.
29. 124.	30. 4·40.		

EXERCISE 57.

Answers given to 3 significant figures.

1. 7·56 sq. in.	2. £40·7.	3. 3·01°/ₒ.	4. £18·8.
5. 5·96.	6. 1090.	7. 5·09.	8. 3·47.
9. 45·3.	10. 3·05.	11. 347 m.p.h.	12. 2·22 seconds.
13. 23·3 feet.	14. 40·2.	15. 558.	

EXERCISE 58 (*a*).

1. $x=\frac{4}{3}$.
2. $(12d-x)$ shillings.
3. 1.
4. $2x^2=(x+1)(x-1)$.
5. $3(x-2)=x(x-1)$.
6. $\dfrac{3x-2y+5}{6}$.
7. $(x+1)(x+2)$.
8. $x=-\frac{5}{2}, \frac{2}{3}$.
9. $12x$ dozen.
10. $x^2=40+x(16-x)$.

EXERCISE 58 (*b*).

1. $x=5, y=-7$.
2. $(3a+4)(2a-1)$.
3. $-\dfrac{13}{17}$.
4. $\dfrac{a^3-b^3-c^3}{abc}$
5.

$x=$	$4\frac{2}{5}$	$1\frac{3}{5}$	$-1\frac{1}{5}$	-4
$y=$	-2	0	2	4.

6. 8·1.
7. $\dfrac{5c-10b-a}{6}$ or $\dfrac{5}{6}c-1\dfrac{2}{3}b-\dfrac{1}{6}a$.
9. $x=-4, 2$.
10. A £10; B £30.

EXERCISE 59 (*a*).

1. $a=49$.
2. $\frac{4}{x}$ hours.
3. $ax=5$.
4. $6x^2=(x+1)(x-1)$.
5. $\frac{1}{x-2}=\frac{2}{x-3}$ or $x-3=2(x-2)$.
6. $x=\pm\frac{9}{2}$.
7. $4x(x+2)$.
8. $x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2=\frac{37}{16}$.
9. $\frac{5b}{a}$ pence.
10. $x(x+1)=30$.

EXERCISE 59 (*b*).

1. $x=7, y=2$.
2. x^2+x-12.
3. $\frac{19-x^2}{5x}$.
4. $a=4$ or 1.
5. $6x^3+x^2-21x+9$.
6. $240x+60y+6z$ pence.
7. $x=-1$.
8. $x=7, y=-2$.
9. $\frac{(a+b)^2}{a^2+b^2}$.
10. 144 eggs.

EXERCISE 60 (*a*).

1. $x=-\frac{8}{3}$.
2. $\frac{mt}{60}$ miles.
3. $y=\frac{4x-8}{7}$.
4. $12x^2+29x+14=6x^2-19x+15$;
or accept $(3x+2)(4x+7)=(2x-3)(3x-5)$.
5. $\frac{4(5x-2)-3(4x-7)+12(x-2)}{12}$.
6. $\frac{1}{2x+3}$.
7. $(x+5)(x+4)$.
8. $x=-1, -2, -3$.
9. £xyz.
10. $x(13-x)=40$.

EXERCISE 60 (*b*).

1. $s=-\frac{1}{2}$.
2. $(5x-3)(2x+1)$.
3. x^2+5x+9.
4. $2a^2+2ab-4b^2$.
5. $2x-3, 2x-1, 2x+1$; 19.
6. $\frac{7}{13}$.
7. $x=8, y=6$.
8. $\begin{array}{llllll} x= & -2 & -1 & 0 & 1 & 2. \\ y= & 10 & 5\frac{1}{2} & 2 & -\frac{1}{2} & -2. \end{array}$
9. $s=25$.
10. 24 miles per hour.

EXERCISE 61 (*a*).

1. $x=\frac{2}{7}$.
2. $\frac{x}{y-1}$ yards.
3. $x=84$.
4. $\frac{2x^2}{(x+y)(x-y)}$.
5. $3(3a-2)-5(2a-3)=15(a-1)$.
6. $x-4y=7$.
7. $(x-12)(x+7)$.
8. $x=\frac{1}{2}$ or $-4\frac{1}{2}$.
9. £$\frac{41}{40}x$.
10. $3x=4(3-x)$.

EXERCISE 61 (*b*).

1. $7y^2-28xy-18x^2$.
2. $2x-7y$.
3. $x=5$ or $-\frac{15}{7}$.
4. $\begin{array}{llllll} t & -2 & -1 & 0 & 1 & 2 \\ s & -2 & -\frac{1}{3} & 1\frac{1}{3} & 3 & 4\frac{2}{3}. \end{array}$
5. $\frac{3bc-2ac+6ab}{5abc}$.
6. $\left(\frac{x}{2y}+\frac{x}{2z}\right)$ hours.
7. $x=\frac{2}{13}$.
8. No real roots.
9. $x=5$.
10. 40 years.

EXERCISE 62 (*a*).

1. $x=1$.
2. $a+b$.
3. $2\frac{1}{2}$ square inches.
4. $\dfrac{1}{x(x+1)}$.
5. 4.
6. $x=-12$.
7. x^3.
8. $x=0$ or $-\frac{7}{5}$.
9. $x+2y$.
10. $x^2=25+12(75-x)$.

EXERCISE 62 (*b*).

1. $5a^2-8ab-13b^2$.
2. $(7x-2y)(3x+5y)$.
3. $x=5{\cdot}74$ or $-1{\cdot}74$.
4. $x=5$.
5. $x=-3$.
6. $x=4\frac{1}{5}$ or -3.
7. $7(x^2+4-y^2)$.
8. $x=1$ or -3.
9. $\dfrac{5x-10}{x-5}$.
10. $6d.$ per lb.

EXERCISE 63 (*a*).

1. $x=3$.
2. $2b$.
3. 3.
4. $-3s-12t+6s-4t-5s-15t$.
5. $\dfrac{x^2+y^2}{x^2-y^2}$.
6. $y=\dfrac{5x+3}{7}$.
7. $2(y-3)$.
8. $6a+b$.
9. $a+b+c$.
10. $\dfrac{1}{x}+\dfrac{1}{x+2}=\dfrac{5}{12}$.

EXERCISE 63 (*b*).

1. $28x^2+4xy+8y^2$.
2. $x=5$, $y=6$.
3. $1\frac{23}{72}$.
4. $x=-\frac{12}{7}$.
5. 10 inches.
6. $x=-\frac{4}{15}$.
7. $x=2$, $y=1$.
8. $14{\cdot}4$; $10{\cdot}404$; $24{\cdot}804$.
9. $A=1$, $B=-1$.
10. $x=2$.

EXERCISE 64 (*a*).

1. 2.
2. $x=2$.
3. $\frac{1}{2}$.
4. $3+\dfrac{2}{x+2}$.
5. -1.
6. $x=\dfrac{7y-5}{3}$.
7. $9l^2-30lm+25m^2$.
8. $x=-\dfrac{7}{5}$ or $\dfrac{8}{3}$.
9. $\dfrac{x+y}{2}$ miles per hour.
10. $\dfrac{x}{y}=\dfrac{1}{2}$; $\dfrac{x-2}{y-1}=\dfrac{1}{3}$.

EXERCISE 64 (b).

1. $4x^2-2x$.
2. $x=8{\cdot}6$ or $-{\cdot}6$.
3. $a=4$.
4. $\dfrac{2x^2+x}{(x+2)(x-2)}$.
5. $3x^2$.
6. $x=\dfrac{7}{20}$.
7. $4x-2y+z$.
8. $x=6,\ y=6$.
9. $x=\frac{5}{3}$ or -1.
10. 250 miles.

EXERCISE 65 (a).

1. $x=\frac{1}{2}$.
2. $a+b$.
3. 22.
4. $\dfrac{2x}{(x+1)(x-1)}$.
5. $10x+9y=30$.
6. A square.
7. $-9x^2+30xy-25y^2$.
8. $2x-3$.
9. $5x$ pence.
10. $\dfrac{24}{x-1}-\dfrac{24}{x}=2$.

EXERCISE 65 (b).

1. $4x^3-20x^2+33x-18$.
2. $\dfrac{x-1}{x+1}$.
3. $a=10\frac{3}{8}$.
4. $x=-2 \quad 1 \quad 4 \quad 7$
   $y=\ \ 10 \quad 5 \quad 0 \quad -5$.
5. $x=2{\cdot}70$ or $-{\cdot}37$.
6. $\dfrac{x}{y}+\dfrac{y}{x}$ shillings; or $\dfrac{x^2+y^2}{xy}$ shillings.
7. $8z-4y-2x$.
8. $4a^2+ab-b^2-bc$.
9. $x,\ 2x,\ 3x+6,\ x+2,\ 2$.
10. 40 men.

EXERCISE 66 (a).

1. $\dfrac{b+a}{b-a}$.
2. $m+n$.
3. 8.
4. $6c-12d-6e+8f$.
5. $x=\frac{1}{4}$.
6. 6 units.
7. $10a^2-21ab-49b^2$.
8. $x=\dfrac{3}{2}$ or $-\dfrac{3}{7}$.
9. $\dfrac{300}{x}$ minutes.
10. $\dfrac{x}{4}=2\dfrac{1}{2}+\dfrac{x}{5}$.

EXERCISE 66 (b).

1. $x={\cdot}1,\ y={\cdot}3$.
2. $x=-{\cdot}5$ or 2.
3. $\dfrac{x}{30}$.
4. $2\dfrac{2}{3}$.
5. $(11x-3)(3x+1)$.
6. $x=9$.
7. $x=6\frac{1}{6}$.
8. $4a^2-2ab$.
9. $\dfrac{a^2-2b^2+3}{a+b}$.
10. 10 feet, 5 feet.

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